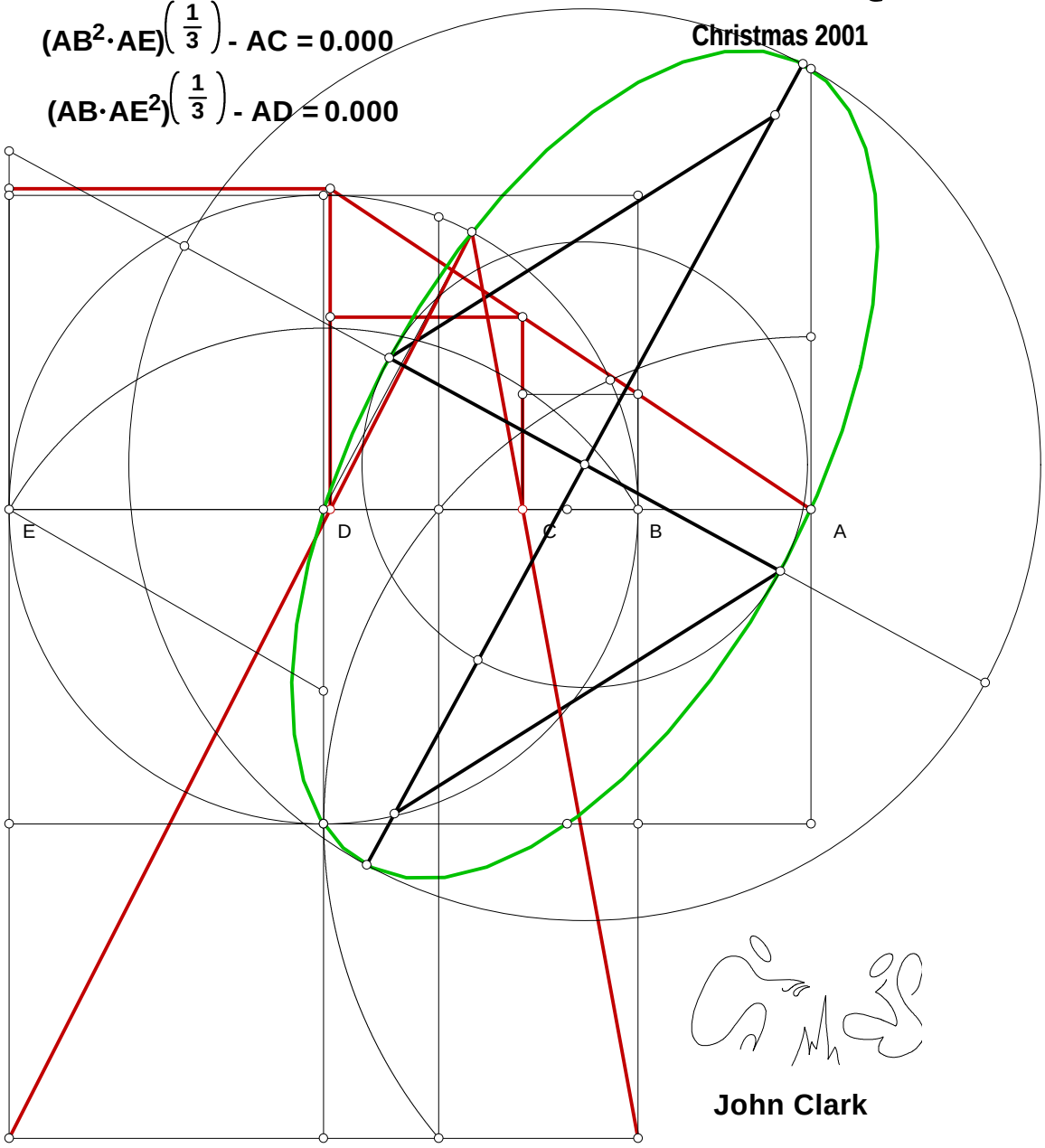
The Delian Quest

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$

Christmas 2001



John Clark

Foreword

One can deny a convention and thus deny a concurrence afforded by that convention. For example, if one wanted to prove that a house could not be built from a set of plans, simply deny the convention of units from which measure is effected. Does this denial afford a proof of the non-existence of the ability to construct that house from a certain set of plans?

If one denies a convention, one cannot solve for the Name Of The Beast. If one denies a convention, one cannot solve The Delian Problem. Solutions are then only for those who have a certain respect for The Law Of Reciprocity–those who aspire to have but one measure in their purse, those who seek to perceive by a single eye–those that that seek the attainment to a civil human state.

There have been hard drive failures and viruses that have claimed some work–sometimes many months worth. A few plates are still in folders, waiting for attention, they have not made it in this text, perhaps they will in a future revision. A sequel, ***Three Pieces Of Paper*** is underway and is on angle trisection. There is so much I want to do, so much I am doing, that revision is inevitable and completions rare.

The Delian Quest

> "Socrates: And I, Meno, like what I am saying. Some things I have said of which I am not altogether confident. But that we shall be better and braver and less helpless if we think that we ought to inquire, than we should have been if we indulged in the idle fancy that there was no knowing and no use in seeking to know what we do not know;—that is a theme upon which I am ready to fight, in word and deed, to the utmost of my power." ***Meno***, by Plato

Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. These two languages are commensurate with the definition of a thing.

Definition: A thing is any difference what so ever circumscribed by any form what so ever.

Commensurate with difference in the definition is the ***Geometric Grammar***. Commensurate with the form in the definition is the ***Algebraic Grammar***. The smattering of English is of no great consequence. I take exception to any teaching that does not recognize the geometric figure as ***the Grammar of Geometry***. Much of what is called geometry is apparently stated by those who have no clue. The ***Geometric Grammar*** system is a ***relatiologic*** and ***Algebraic Grammar*** is a ***tautologic***–together they make a proposition–and together and only together can a ***thing*** called ***knowledge*** be constructed–by definition.

By way of definition, so called non-Euclidean Geometries which cannot construct the figures is not knowledge and those that use Euclidean figures but not standard definitions and practices are also not knowledge. Since the ***Geometric Figure*** is the ***Geometric Grammar***, there is no such thing as a Grammar without Grammar. And by the principle of well-defined terms, not knowing how to define a thing, or the inability to be consistent in what has been defined, does not constitute literacy.

The Delian Problem

From what I remember, the Delian Problem acquired its name from the circumstances of the problem's inception. In order to stop a plague in ancient Greece an oracle at Delos suggested–supposedly by direction of a *God*–that the altar of Apollo be doubled. The altar of Apollo was a cube. What it boiled down to was the abstraction of the cube root of 2–the cube root of twice its current volume. Now if, in geometry, one can precisely abstract the square root of 2, it seemed reasonable that, in geometry, one might abstract the cube root of 2 as well. What this means is that an act of will may be performed for which no Arithmetic name is possible in the case of the square maybe it can be done for the cube as well. The conventions of Arithmetic Grammar cannot provide an Arithmetic name for the square root of 2. Algebraic grammar overcomes this problem of irrationality in Arithmetic Grammar by incorporating operands as part of the Algebraic name. The problem in Geometric Grammar was, no one knew how to abstract cube roots. No one knew how to assert that act of will. In time, the general response to the Delian Problem was that it is an impossible problem to solve. No name could be provided in Arithmetic Grammar, and none in the Grammar of Geometry, while in Algebraic grammar, a name

could be simply had, but no clue as to how to render the fact.

A Class Idea

Those things that are grouped in accordance by a common characteristic all have the same name by that measure. Some people may recognize this notion as Set Theory, some may see it as the founding principle of all Grammars because it is the foundation upon which the concept of words are based. A fact of the craft of measurement itself is derived from $A = A$ and $A - A = 0$ which is derived from knowing and not knowing, some may realize that it is the ***Sensor Model*** of reality. This has a very important application. If there is no difference between two things, then there is no knowledge of 'two-ness' under the class definition–therefore both items are regarded as the same–treated the same, respected the same. By definition, it is not possible to act differently in any case or any instance toward members of the same set. Action is a reaction from cause, but one has stated that all members are the same, no difference or cause can then exist between them–under that class definition. The definition of a set provides Universality circumscribing all members of that set.

In regard to the Delian problem one might apply the idea of definition in this manner; take the straightedge and compass and abstract a common characteristic that would group them into one class (to my knowledge such an application of the foundation of logic has never been proposed to solve the Delian Problem). If one finds that common characteristic then one can use it as a measure to determine that other tools, which have that same characteristic, are included in that class. Perhaps these tools are what is needed to solve the Delian Problem. Perhaps the real intention of the problem was to learn to think in terms of definition–a founding principle of judgment.

The tool one would be looking for would have the class characteristic that would define a Two Dimensional Geometer's Writing Instrument defined by a common characteristic. This is the true custom upon which all language is based, the same notion that determines the meaning of any word in any language that one speaks. Logically, reasonably, factually, one cannot state that Geometry is to be written in only two tools of a class of tools if that class is well defined, for there is no way to distinguish these tools under the class concept, the definition of class membership. Providing of course one can abstract the class concept that the tools reside in. Aristotle quite rightly determined that a definition renders both ***substance*** and ***form***–every bit of knowledge is known that way, ***substance*** and ***form***. Even in our mind, to think of ***substance*** we supply a generic ***form*** to it, and to think of ***form*** we assert a generic ***substance*** for it–otherwisde we could not conceive of either. What is the ***substance*** and the ***form*** of the Geometric Writing Instruments we are familiar with?

A straight edge provides one with one and only one difference (***substance***) between two points (***form***). A compass provides us with one and only one difference (***substance***) between two points (***form***). One may comprehend the straightedge however as providing the Unit of discourse, while the compass the Universe. Has anyone ever comprehended the ellipse as a figure that is produced as one and only one difference (***substance***)–a sum–between two points (***form***)? Is it possible that the *Gods* gave a problem, just like the problem of understanding the ***Name Of The Beast*** in the ***Bible***, whereby the only solution is to think by one measure–class concepts–the very foundation of language

itself? Perhaps the only possible path to a solution is when men start to reason by respecting what, as recorded in the Bible, is a Holy understanding–the ***Law of Reciprocity***? ***A*** = ***A***. I don't know. I do know that the tool that constructs an ellipse is in the same class as the straightedge and compass in regard to the same definition of ***substance*** and ***form***, and also that the ellipse can solve the Delian Problem. I would strongly suspect that the problems given by the *Gods* all have a common solution which involves a growth in men's understanding. I did not suspect such an understanding when I started my search. I started this search because either no *God* gave the Delian Problem or ,if given, it would seem reasonable that a *God* would only ask for the possible and, if it were possible, then it was possible for me to find it.

Proof And Prove

Proofing and proving are two entirely different concepts. Proofing is the compliance of conventions within a grammar. Proving is the verification of existence–which means attaining to an experience.

The proof of a proposition in a tautologic is then relies on a tautology, ***name*** = ***name***. One might have a name, but no meaning of that name, but still attain to proof. Proof in a relatiologic is constructions tendered through a given set of tools used in a prescribed manner. One might have a drawing with no meaning, but still it is proofed.

It is typical even for scholars, whole communities and histories of them, to confuse and make a mess out of the two ideas. To prove a thing, one must supply both logic systems, the tautological and the relatiological. To proof a thing, one needs only demonstrate compliance with the conventions of the grammar. Proving implies proofing, but proofing does not imply proofing–Aristotle said very nearly the same thing.

To prove a thing, is to tender both the experience and the names in compliance with a given set of conventions–however, one cannot say that one has attained to conventionality by listing those conventions in the Particular, avoiding or not knowing the Universal upon which they stand–matter and form must be known and knowable.

Proposition

Even the process of group construction (propositional construction) follows the definition of proposition itself. One method is particular (the ***substance***), each member is enumerated, the other method is universal (the ***form***), each member has the same definition. Each method renders a judgment as to inclusion and exclusion of a member into a class. As Plato pointed out, only by comprehending all the details of both methods can one be said to have understanding. The methods themselves follow propositional form in another respect–the particular method is not effected through understanding but the method of forms (definition) is–this Plato pointed out too and is why he championed that method. Using class definition to define Geometric Writing Instruments is an idea that has not been promoted or conceived, and if enacted many mistakes will be made, but nevertheless, we may not be altogether certain as to what we are doing, but we can be certain that life is in that quest for knowing. Perception determines conception, conception determines will.

Reading

Since perception determines conception and conception determines will, learning to 'say' what we 'see' can be improved by practice. One must read the figure and the equations together in this work. j.c.

> "For the person who deliberates seems to investigate and analyze in the way described as though he were analyzing a geometrical construction" ***Nicomachean Ethics*** by Aristotle

The Delian Quest

1989

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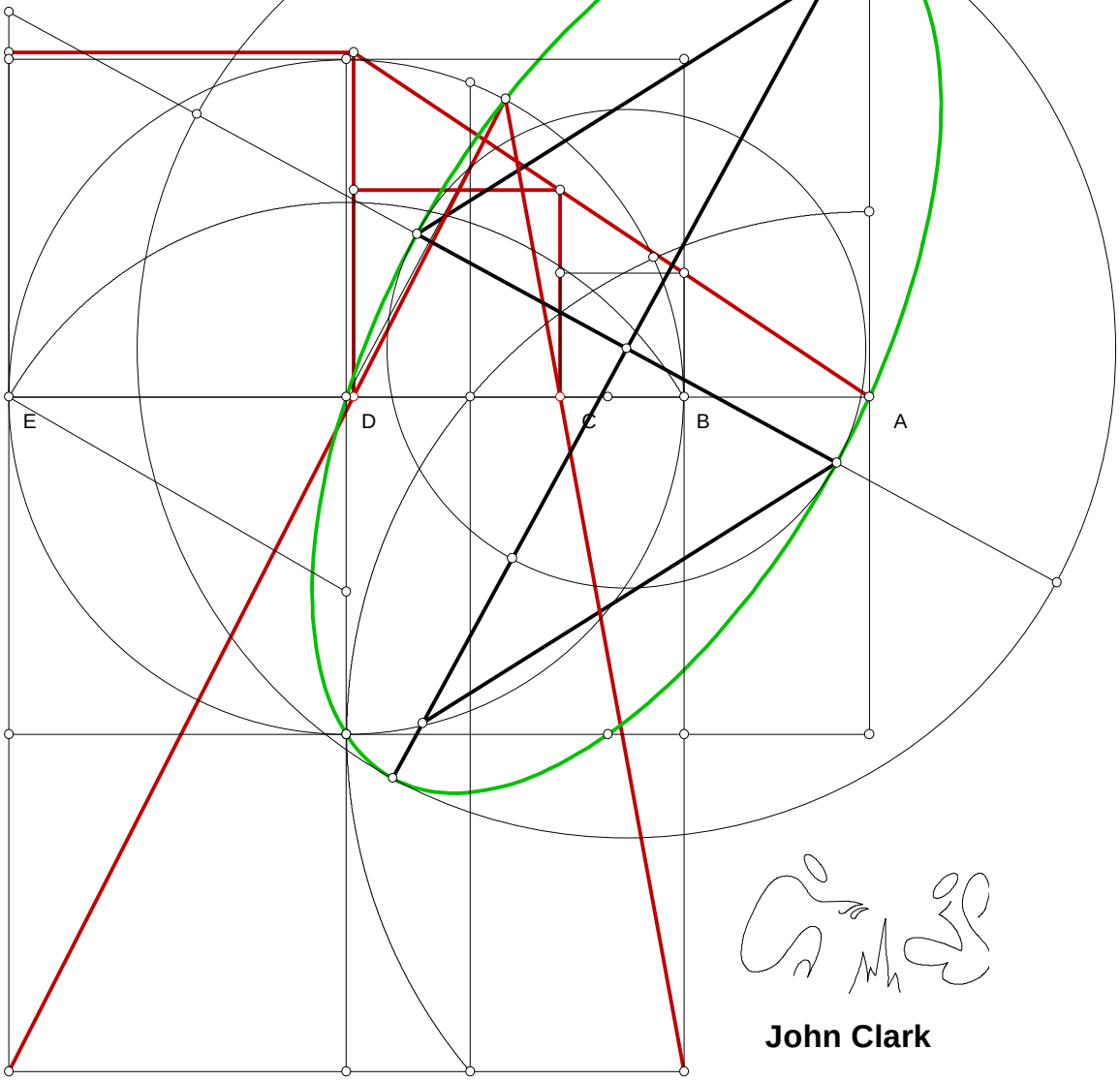


John Clark

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

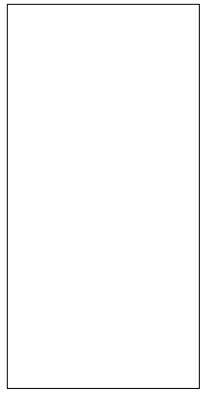


Plate 1

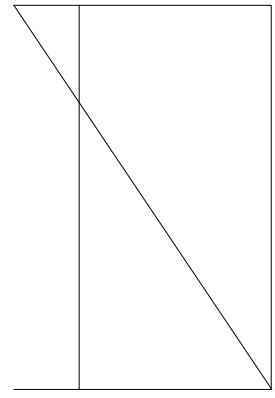


Plate 2

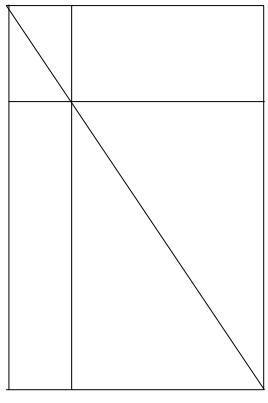


Plate 3

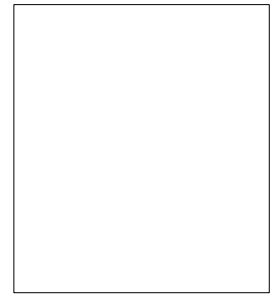


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

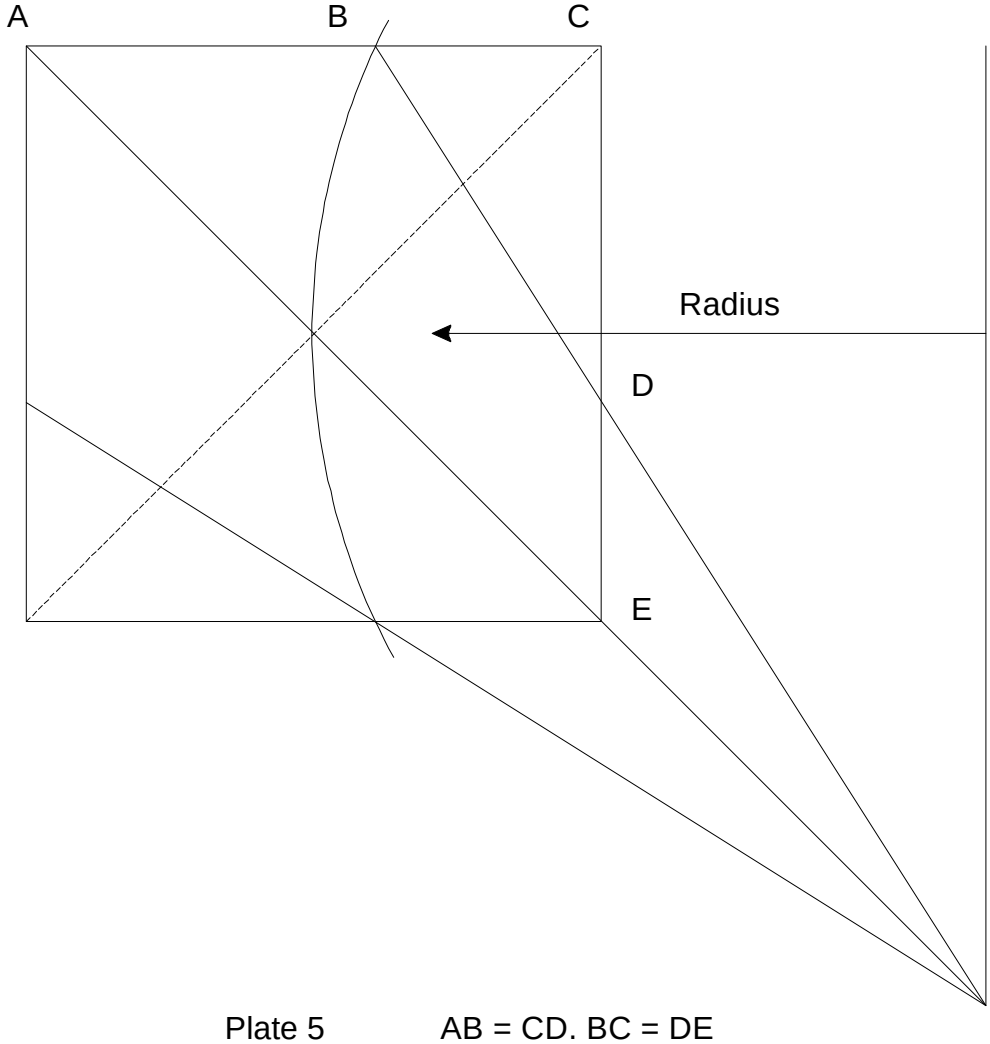
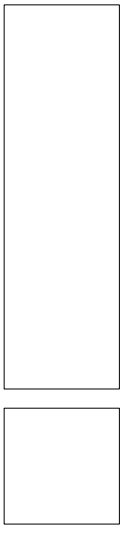
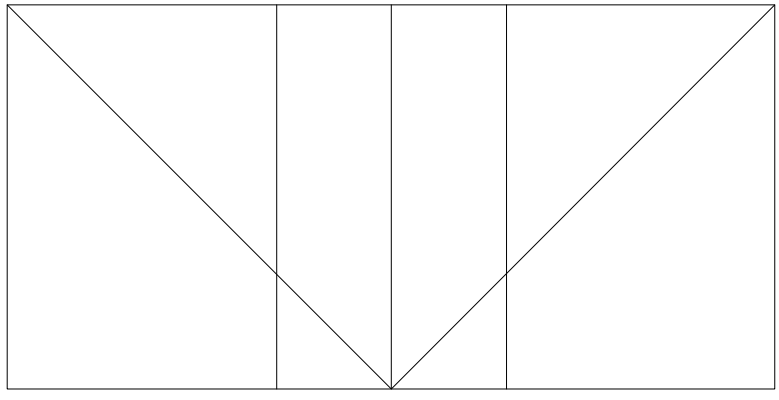


Plate 5 AB = CD. BC = DE

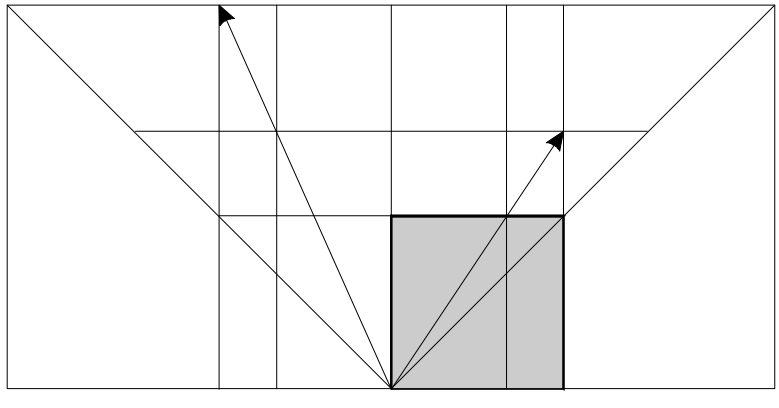
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



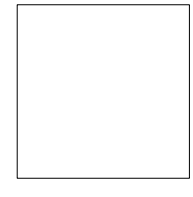
P.6



P.7

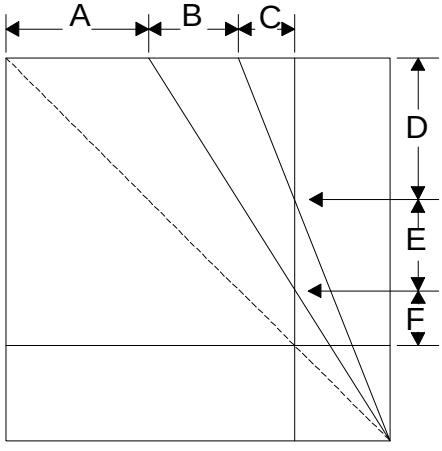


P.8

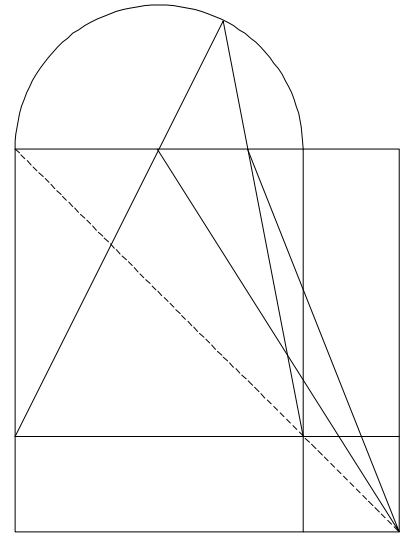


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

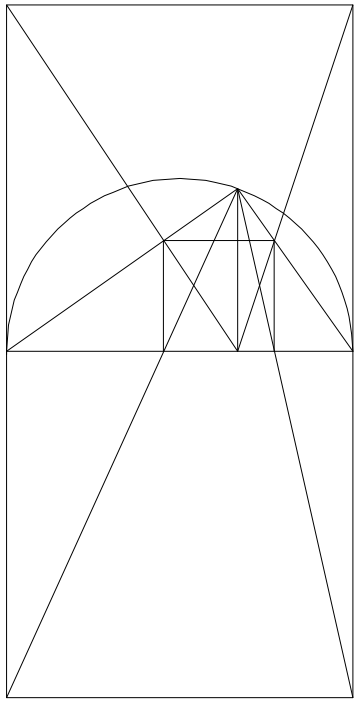


P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

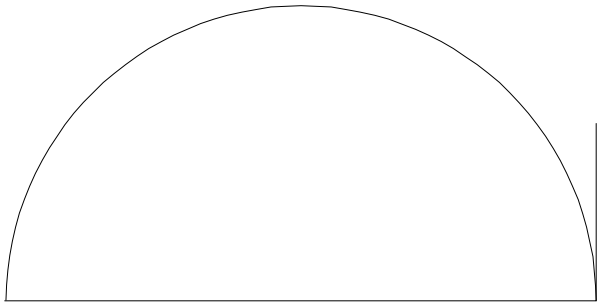


Plate 13

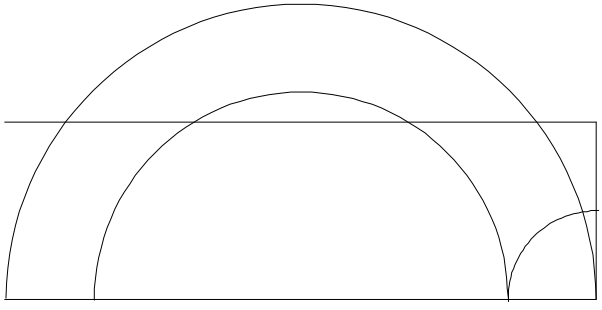


Plate 14

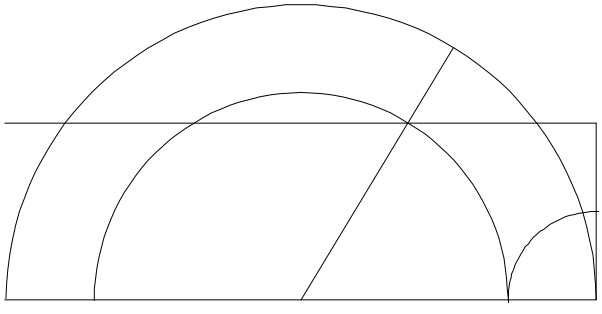


Plate 15

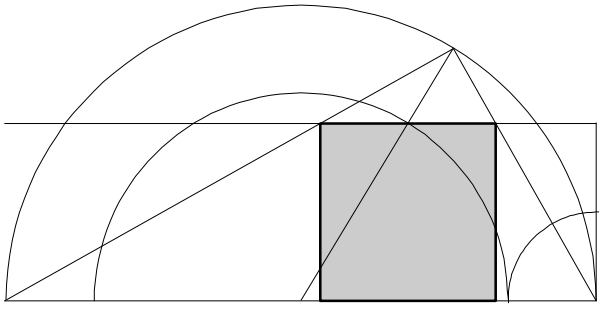
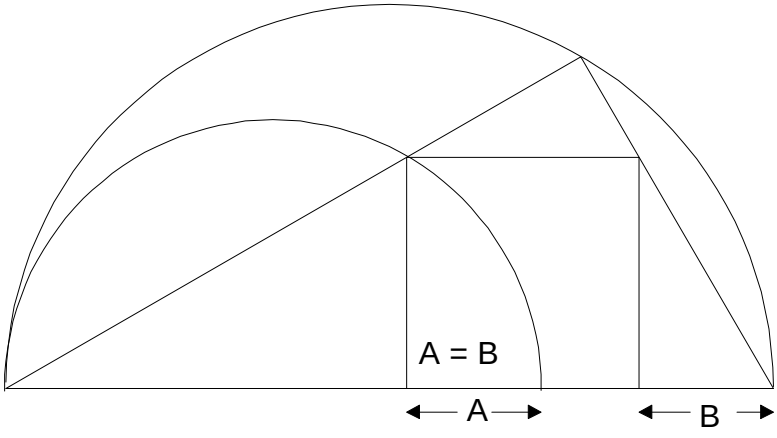
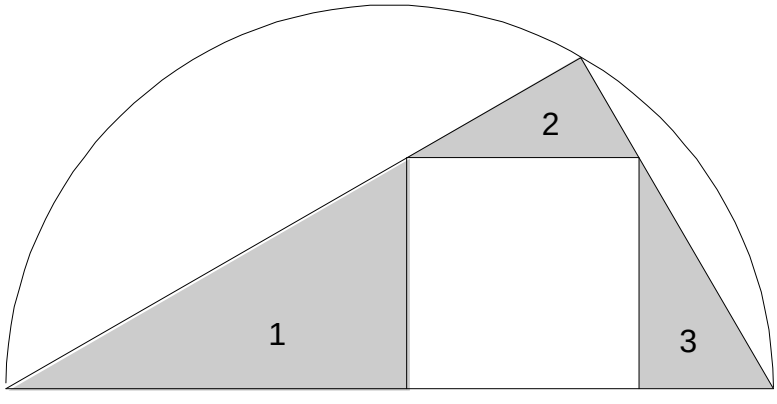


Plate 16

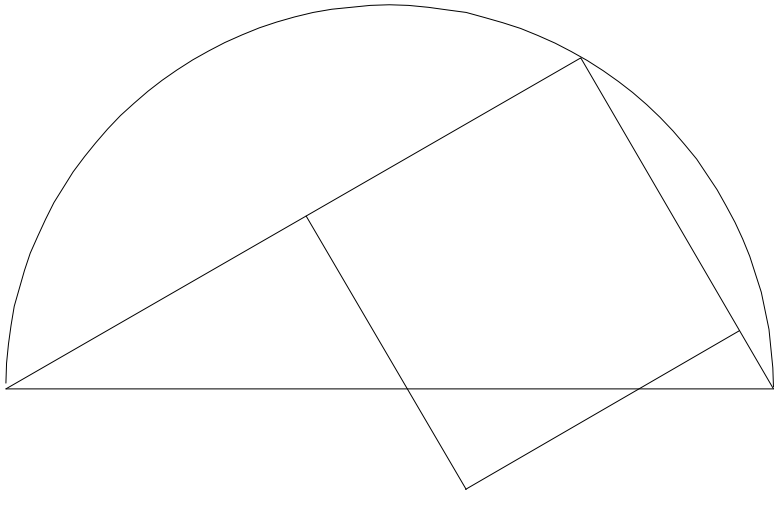
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.



In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"



P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

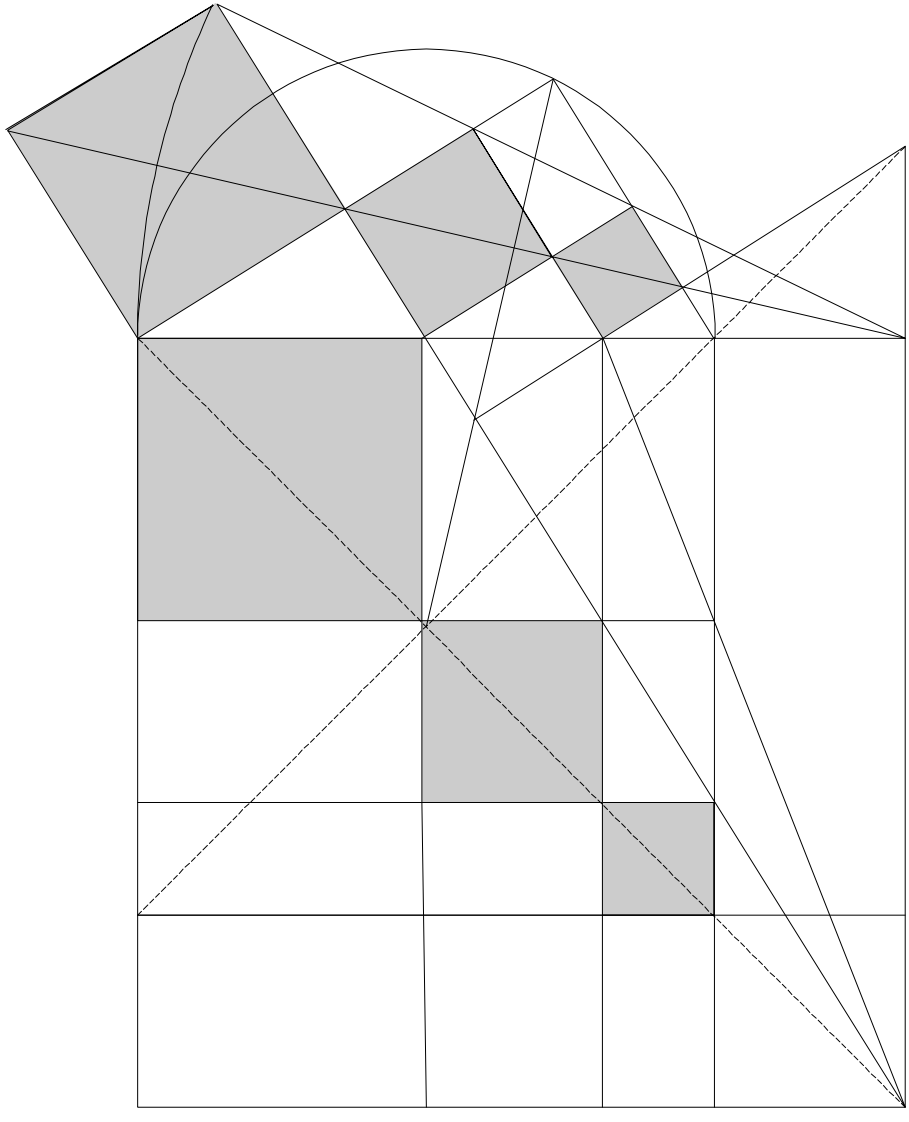
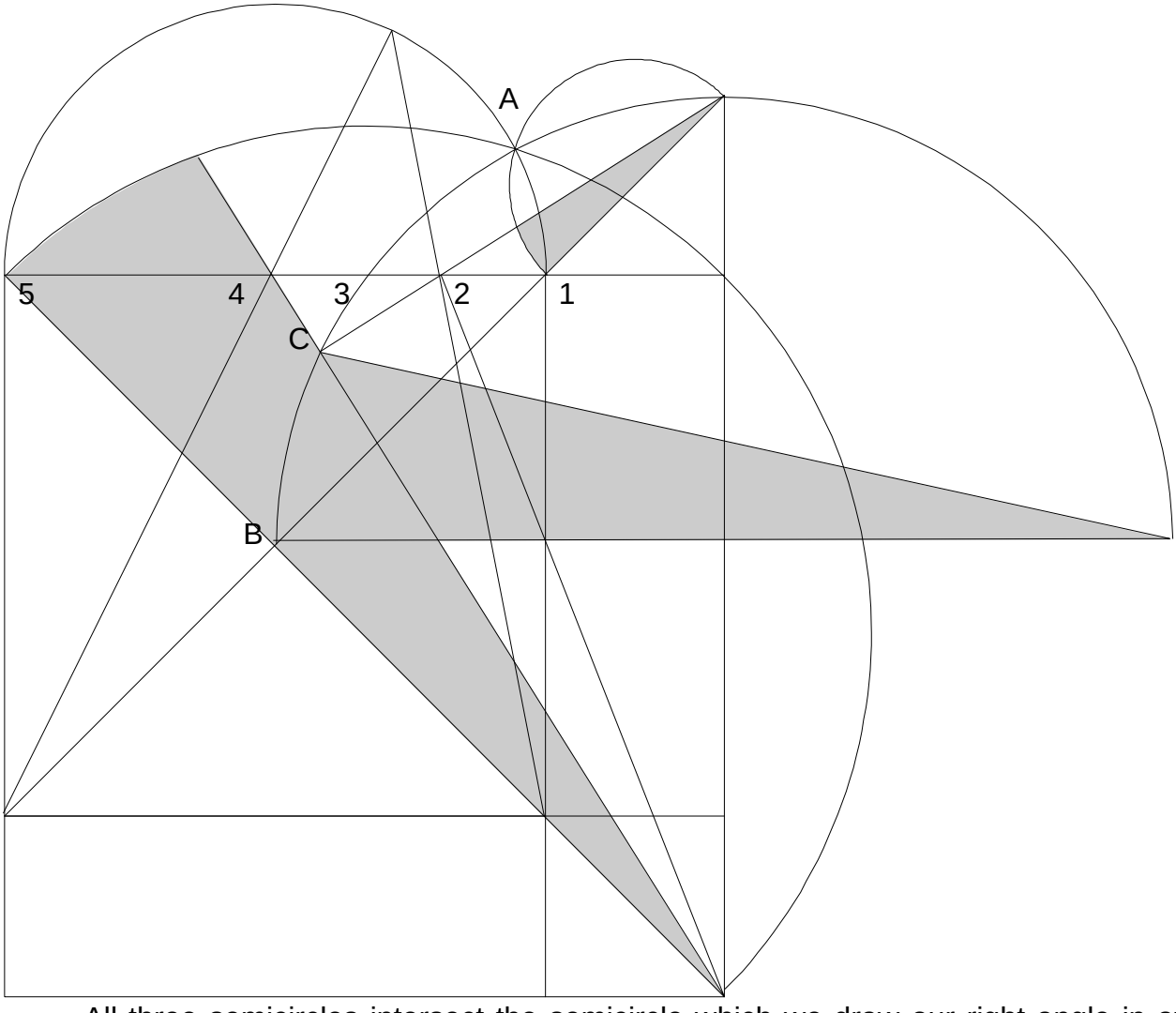


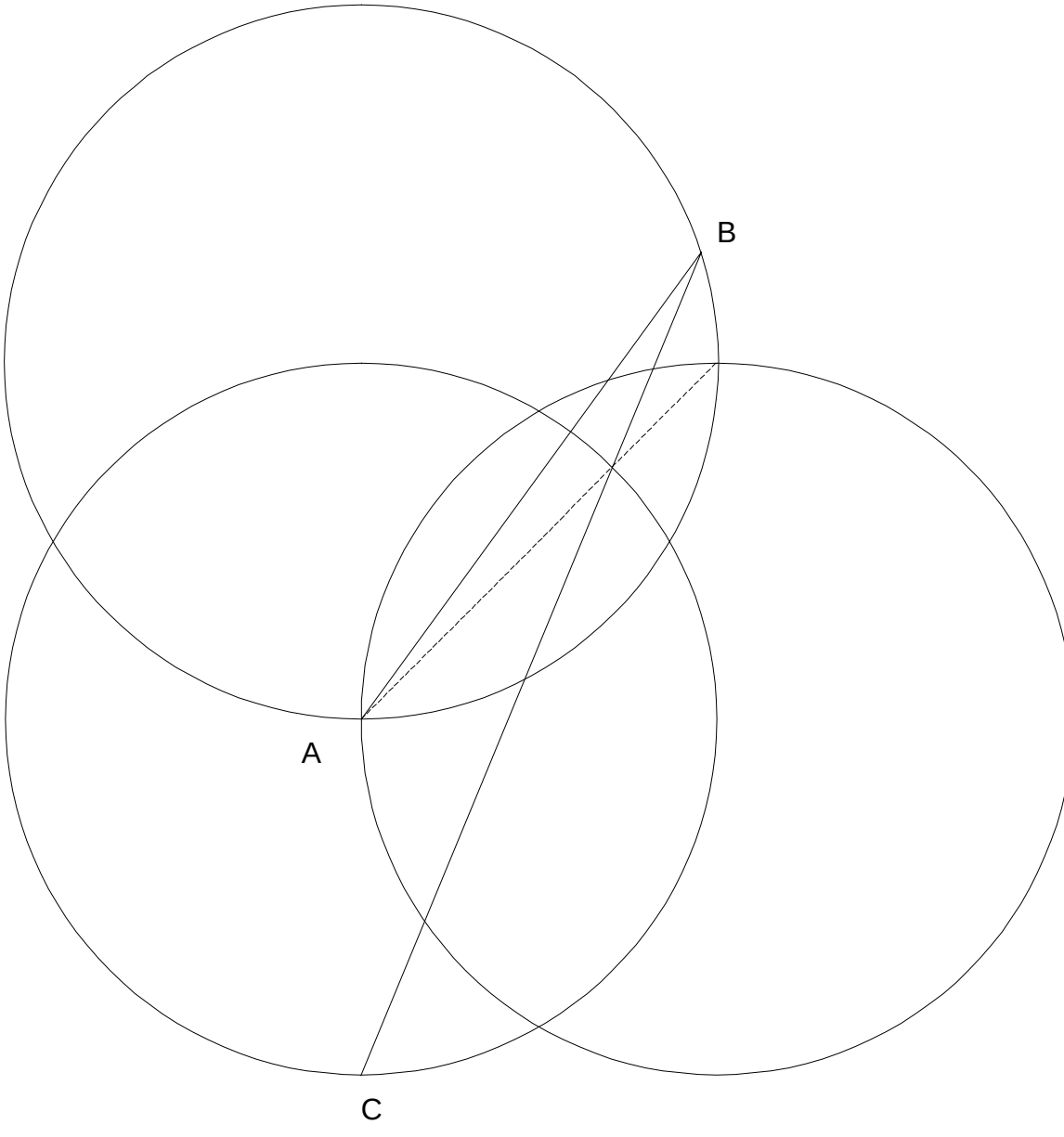
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.



All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.



How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

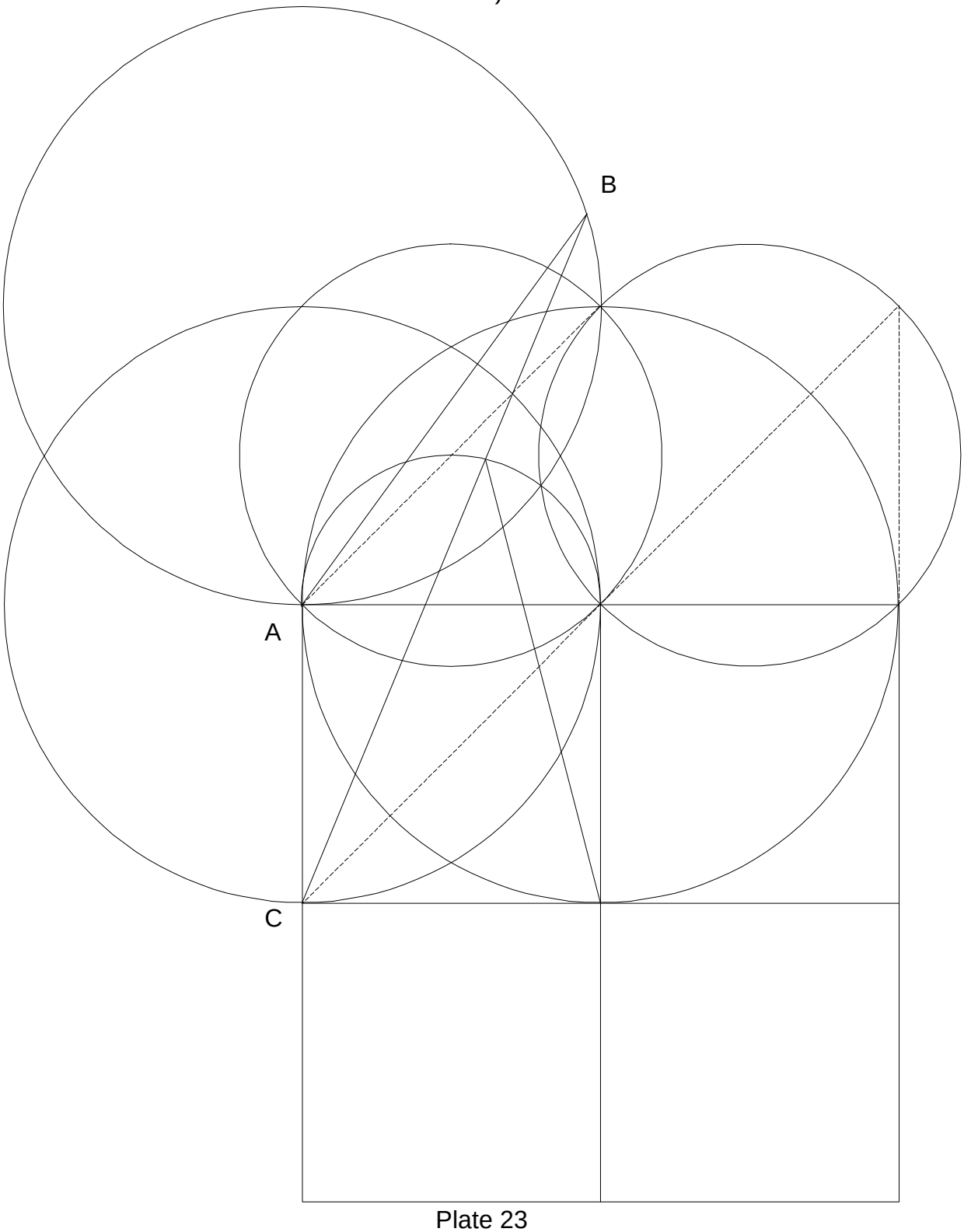
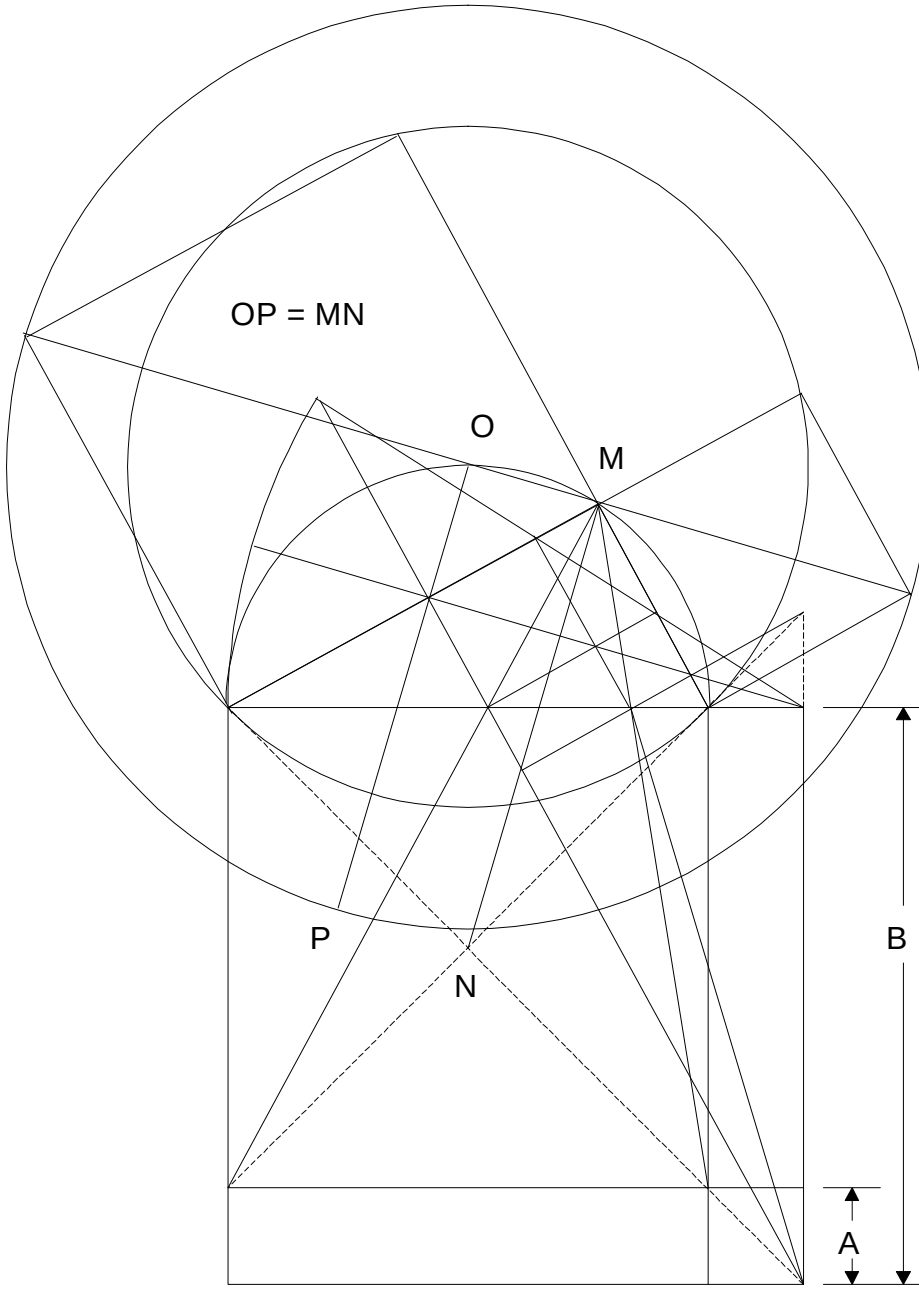


Plate 23

On Plate 24 the radius for the circle OP is given by MN.



One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of B^2A (if you have missed it, the figure gives both roots, A^2B and B^2A) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

GEOMETRIAE DEDICATA

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Telephone: (0)9131-8525(X)
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F. D. VELDKANW
NblemakchMm4uw
Postbus 80.010
3508 TA Utrecht
The Netherlands
Telephone: (0)30-531519

Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

Location:
201 Charles **Street**
Providence, **RI**
02904

December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

Serving the mathematical community for over 100 years

American Mathematical Society

Department of Mathematics
Roger E. Howe
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

société mathématique defrance

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

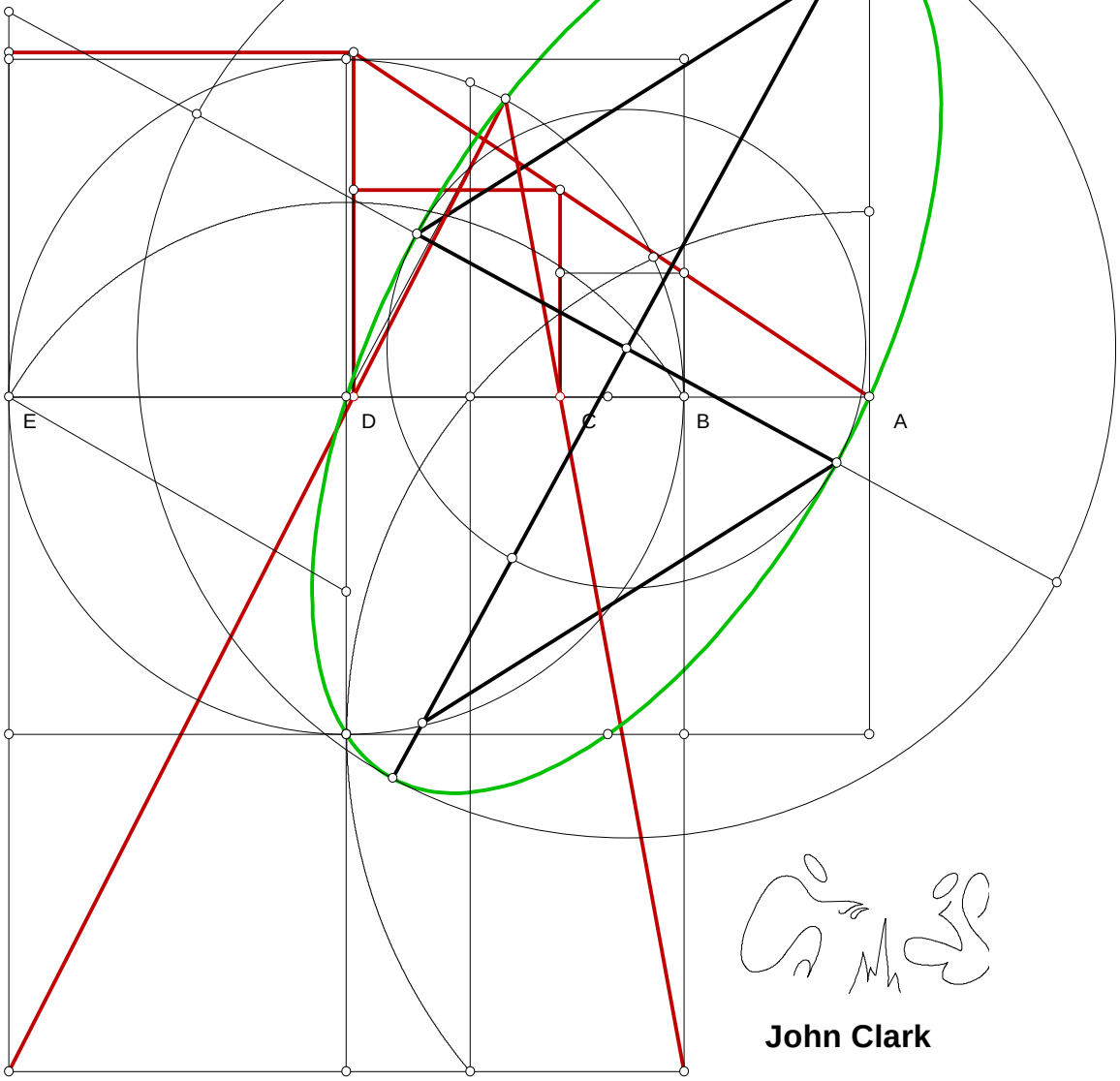
P.J. : Manuscrit

The Delian Quest

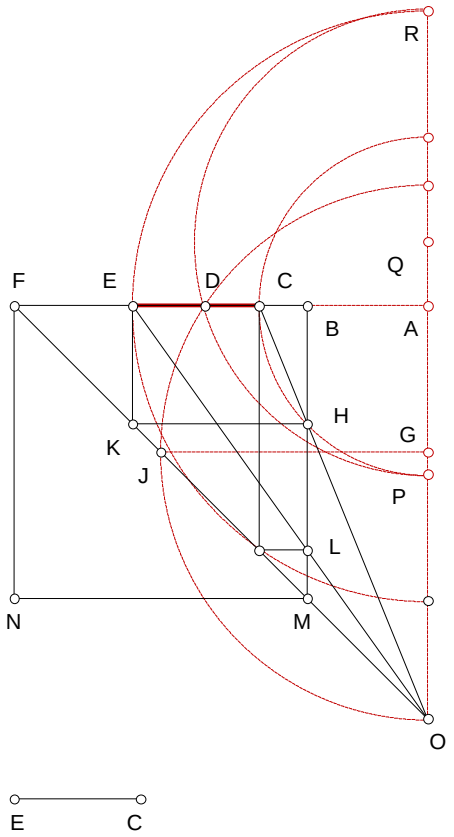
1992

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark



A Triplicate Ratio 06/20/92

Given some point O place CE on BF such that O is the point of similarity.

$N := 1.52 \quad N_2 := .89 \quad N_3 := .66$

$BF := N \quad MO := N_2 \quad CE := N_3$

$FM := \sqrt{2 \cdot BF^2} \quad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF} \quad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \quad QR := DQ \quad QP := DQ$

$AP := QP - AQ \quad AR := QR + AQ \quad AC := AP$

$AE := AR \quad AO := AF \quad BC := AC - AB$

$EF := AF - AE \quad EK := EF \quad BH := EK$

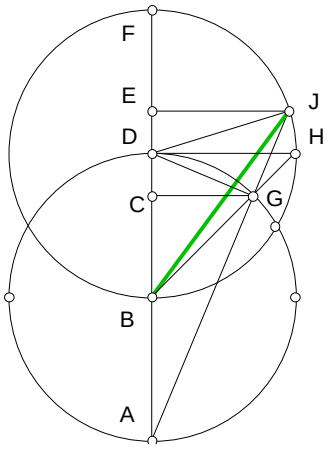
$\frac{BC}{BH} - \frac{AC}{AO} = 0$

The last ratio can be tediously proved by reducing each term to the givens.

Edit 062800

08/12/92 Rusty Cube of a Sphere

Given AB, how close is BJ to the cube root of AB taken as a sphere?



$N := 9$ $\quad CUBE_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

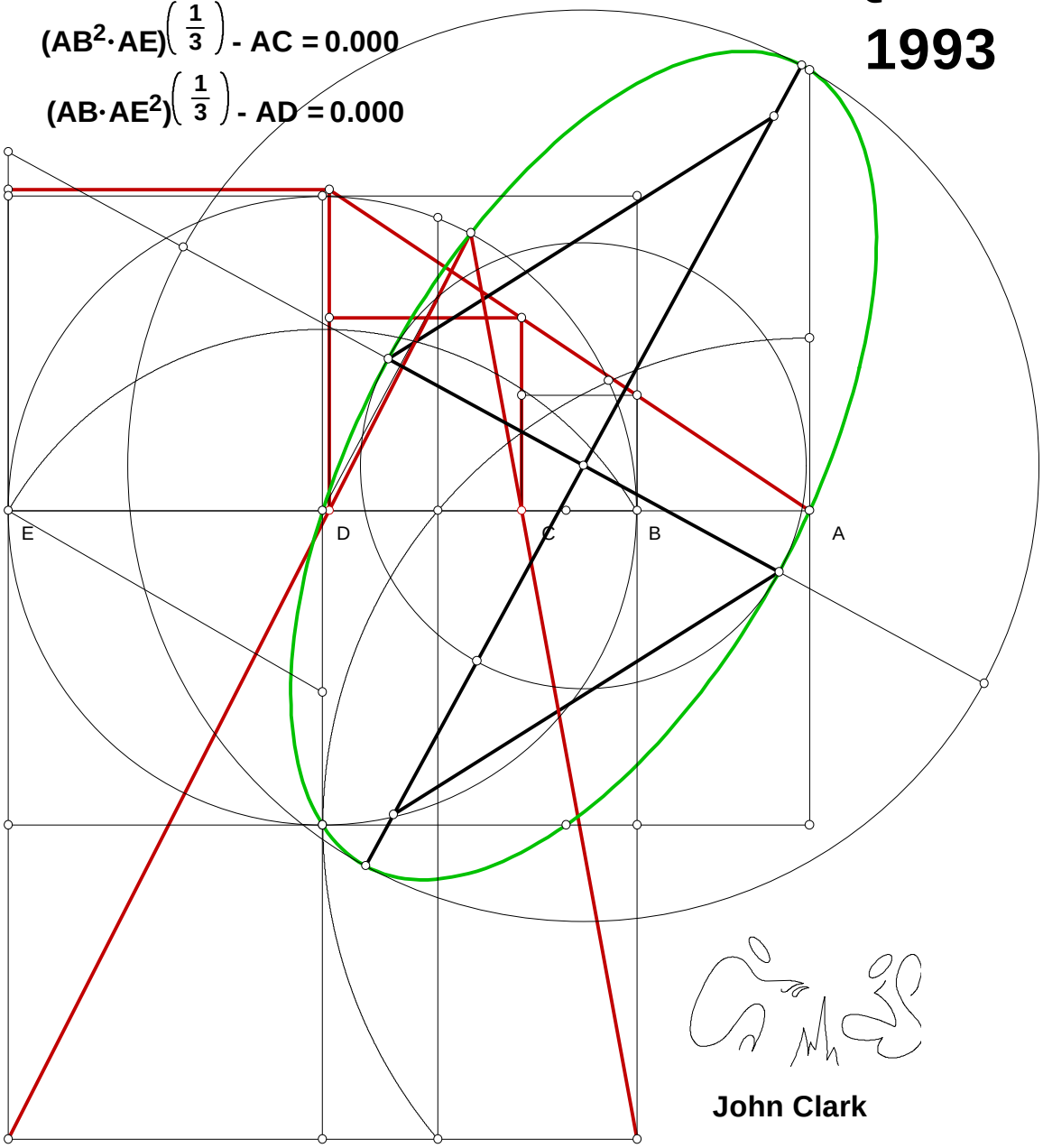
$\frac{CUBE_ROOT}{BJ} = 0.99906$

The Delian Quest

1993

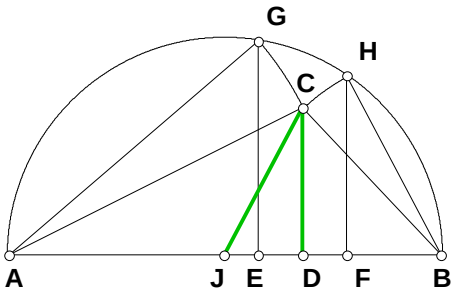
$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

01/08/93 Pythagoras Revisited



Given AB, BC, AC, what is CD, AD, BD and CJ?

$S_1 := 2 \quad S_2 := 3 \quad S_3 := 4$

$AB := S_1 \quad BC := S_2 \quad AC := S_3$

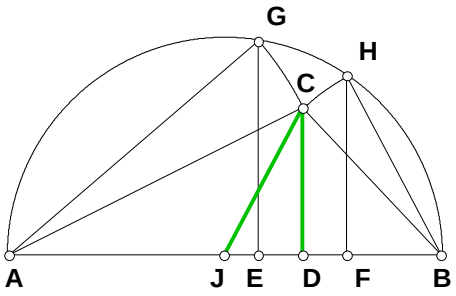
$AG := AC \quad BH := BC \quad AE := \frac{AG^2}{AB} \quad BF := \frac{BH^2}{AB}$

$EF := AB - (AE + BF) \quad DE := \frac{EF}{2} \quad DF := DE$

$AD := AE + DE \qquad AD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$BD := BF + DF \qquad BD := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CD := \sqrt{AC^2 - AD^2} \qquad CD := \frac{\sqrt{(-S_1 + S_2 - S_3) \cdot (S_1 + S_2 + S_3) \cdot (S_1 - S_2 - S_3) \cdot (S_1 + S_2 - S_3)}}{2 \cdot S_1}$



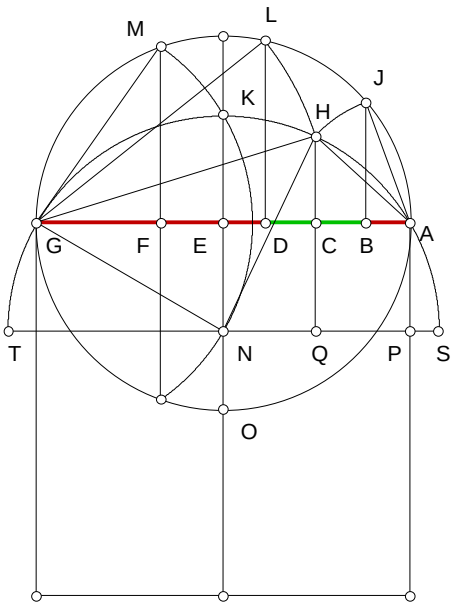
$AJ := \frac{AB}{2} \quad JD := AD - AJ \quad CJ := \sqrt{JD^2 + CD^2}$

$CJ := \frac{1}{2} \cdot \sqrt{2 \cdot S_3^2 - S_1^2 + 2 \cdot S_2^2}$

$\left(AC^2 + BC^2\right) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).

06/03/93.MCD Exploring The Curve CJ



Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?

$N := 2 \qquad N_2 := 4$

$AG := N \qquad AC := \frac{AG}{N_2} \qquad GF := \frac{AG}{3} \qquad AE := \frac{AG}{2}$

$EG := AE \qquad AF := AG - GF \qquad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \qquad GN := GM \qquad EN := \sqrt{GN^2 - EG^2}$

$NH := GM \qquad NS := GM \qquad PN := AE \qquad PS := NS - PN$

$ST := 2 \cdot GM \qquad SQ := AC + PS \qquad QT := ST - SQ$

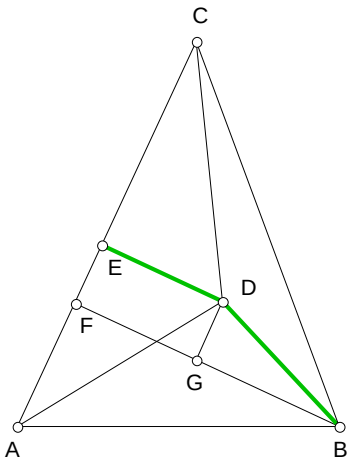
$QH := \sqrt{SQ \cdot QT} \qquad CQ := EN \qquad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \qquad CG := AG - AC \qquad GH := \sqrt{CG^2 + CH^2} \qquad AJ := AH \qquad AB := \frac{AJ^2}{AG}$

$GL := GH \qquad DG := \frac{GL^2}{AG} \qquad BD := AG - (AB + DG) \qquad \sqrt{AB \cdot DG} - BD = 0$

$$\left(\frac{-N}{3} + \frac{N}{3} \cdot \frac{\sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2} \right) - BD = 0$$

06/07/93 For All Triangles Find BD



Given AB, BC, AC, CD, AD, find BD

To simplify use line names found in 010893

$S_1 := 5.55$ $S_2 := 4.61$ $S_3 := 1.5$

$S_5 := 4.5$ $S_6 := 5.48$

$AB := S_5$ $BC := S_6$ $AC := S_1$ $CD := S_3$ $AD := S_2$

$$CE := \frac{CD^2 + AC^2 - AD^2}{2 \cdot AC} \qquad CF := \frac{BC^2 + AC^2 - AB^2}{2 \cdot AC} \qquad EF := CF - CE$$

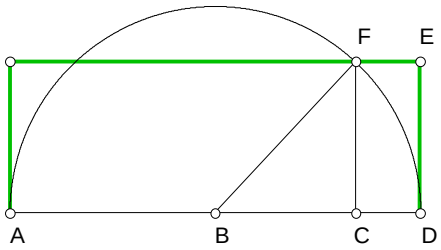
$$DE := \sqrt{CD^2 - CE^2} \qquad BF := \sqrt{BC^2 - CF^2} \qquad GF := DE$$

$$BG := BF - GF \qquad DG := EF \qquad BD := \sqrt{DG^2 + BG^2} \qquad BD = 3.983$$

OR

$$BG_2 := BF + GF \qquad DG_2 := EF \qquad BD_2 := \sqrt{DG^2 + BG_2^2} \qquad BD_2 = 5.757$$

06/09/93 Rectangular Roots



Given AD and DE divide AD into the rectangular roots of DE.

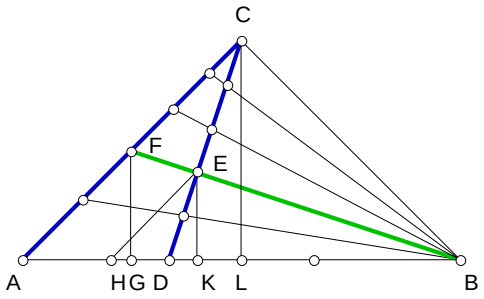
$N := 5 \qquad N_2 := 2$

$AD := N \quad DE := N_2 \quad CF := DE \quad BF := \frac{AD}{2}$

$AB := BF \quad BC := \sqrt{BF^2 - CF^2} \quad AC := AB + BC \quad CD := AD - AC \quad \sqrt{CD \cdot AC} - DE = 0$

$AC := \frac{1}{2} \cdot N + \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2} \qquad CD := \frac{1}{2} \cdot N - \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2}$

06/21/93 Pyramid of Ratios I



Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?

$N_1 := 3 \qquad N_2 := 5 \qquad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AL := \frac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

δ

1
2
3
4
5

$DE_\delta := \frac{CD \cdot \delta}{N_2} \quad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

3.75
5
7.5
15
0

$\text{if}\left(N_2 - \delta, \frac{N_1 \cdot N_2}{N_2 - \delta}, 0\right)$

3.75
5
7.5
15
0

$\frac{AC}{AF_\delta}$

3.667
2
1.444
1.167
1

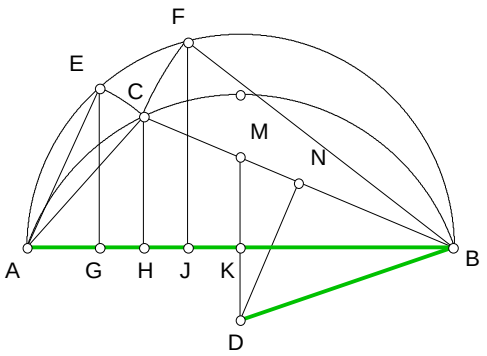
$\frac{(N_1 - 1) \cdot N_2 + \delta}{N_1 \cdot \delta}$

3.667
2
1.444
1.167
1

06/27/93 Describe A Circle About a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $\quad NOT(X) := X = 0$ $\quad \delta := 0 .. 2$



$BK := \frac{AB}{2}$ $\quad AE := AC$ $\quad BF := BC$

$AG := \frac{AE^2}{AB}$ $\quad BJ := \frac{BF^2}{AB}$ $\quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $\quad BH := BJ + HJ$ $\quad CH := \sqrt{BC^2 - BH^2}$

$BN := \frac{BC}{2}$ $\quad BM := \frac{BC \cdot BK}{BH}$ $\quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $\quad BD := \sqrt{BN^2 + DN^2}$

$radius := if(\Delta, BD, 0)$ $\quad imaginary_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary_radius = 0$

$\Delta = 1$

$S_1 := \begin{bmatrix} AB \\ AC \\ BC \end{bmatrix}$ $\quad S_2 := \begin{bmatrix} AC \\ BC \\ AB \end{bmatrix}$ $\quad S_3 := \begin{bmatrix} BC \\ AB \\ AC \end{bmatrix}$

$AB \equiv 3$ $\quad AC \equiv 4$ $\quad BC \equiv 6$

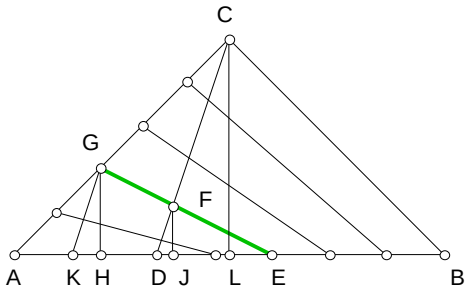
The name of the Radius as a proportion of the given names

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = [\ 3.375 \quad 3.375 \quad 3.375\ ]$

07/15/93 Pyramid of Ratios II

AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?



$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

1.5
2.333
4
9
0

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

1.5
2.333
4
9
0

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

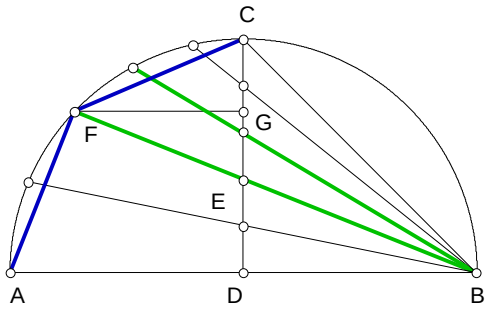
15
4.375
2.222
1.406
1

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

15
4.375
2.222
1.406
1

07/25/93 Pyramid of Ratios III

Dividing DC by an number provides wht in terms of BE/BF and AF/CF?



$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

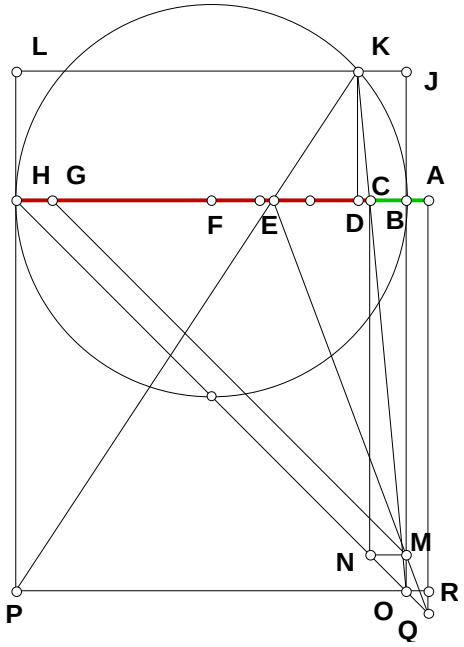
$DG := DE + EG \qquad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

$\frac{DE_\delta}{DG_\delta}$	$\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$	$\frac{BE_\delta}{BF_\delta}$	$\frac{BF_\delta}{BE_\delta}$	$\frac{2}{\left(\frac{\delta}{N}\right)^2 + 1}$
0.514	0.514	0.514	1.946	1.946
0.556	0.556	0.556	1.8	1.8
0.625	0.625	0.625	1.6	1.6
0.722	0.722	0.722	1.385	1.385
0.847	0.847	0.847	1.18	1.18
1	1	1	1	1

$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$	$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$
0.283	0.283
0.707	0.707
1.414	1.414
2.828	2.828
7.071	7.071
0	0

11/06/93 Gruntwork I on the Delian Solution



Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?

$N := 4$ $BH := 1$

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $BJ := DK$ $BO := BH$

$JO := BJ + BO$ $JK := BD$ $CD := \frac{JK \cdot DK}{JO}$

$KL := DH$ $LP := JO$ $DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD$ $CE := CD + DE$ $MN := BC$

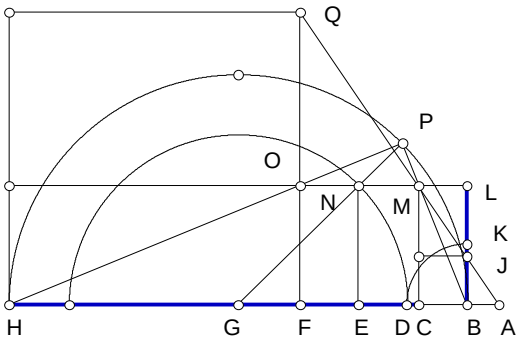
$GH := MN$ $CH := BH - BC$ $HN := \sqrt{2 \cdot CH^2}$ $GM := HN$ $EH := CH - CE$ $EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG}$ $HO := \sqrt{2 \cdot BH^2}$ $OQ := HQ - HO$ $OR := \sqrt{\frac{OQ^2}{2}}$ $AB := OR$ $AC := AB + BC$

$AE := AB + BD + DE$ $AH := AB + BH$ $(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0$ $(AB \cdot AH^2)^{\frac{1}{3}} - AE = 0$

11/09/93 Solve For Cube Root Placement

With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



$N := 2 \quad BH := N - 1$

$BG := \frac{BH}{2} \quad CF := N^{\frac{2}{3}} - N^{\frac{1}{3}}$

$BL := CF \quad GP := BG \quad BK := \frac{BL}{2}$

$BD := BK \quad NP := BD \quad GN := GP - NP$

$EN := BL \quad GE := \sqrt{GN^2 - EN^2}$

$CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD$

$FH := GH + GE - EF \quad FQ := FH \quad FO := BL \quad OQ := FQ - FO \quad MO := CF \quad AF := \frac{MO \cdot FQ}{OQ}$

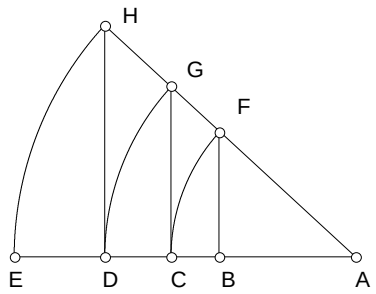
$AC := AF - CF \quad AH := AF + FH \quad AB := AH - BH$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 2$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

11/10/93 Gruntwork II on the Delian Solution

Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?



$N := 4.846$ $AE := 1$

$DE := \frac{AE}{N}$ $AD := AE - DE$ $AH := AE$

$AG := AD$ $AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2$ $\frac{AD}{AB} = 1.588$ $\frac{AC}{AB} = 1.26$

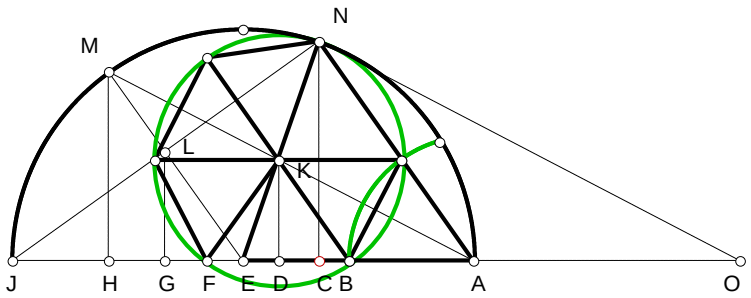
Albebraic Names:

$DE := \frac{1}{N}$ $AD := 1 - \frac{1}{N}$ $AC := \frac{(N-1)^2}{N^2}$ $AB := \frac{(N-1)^3}{N^3}$

$\left[\left[\frac{(N-1)^3}{N^3}\right]^2\right]^{\frac{1}{3}} - \frac{(N-1)^2}{N^2} = 0$ $\left[\frac{(N-1)^3}{N^3}\right]^{\frac{1}{3}} - \left(1 - \frac{1}{N}\right) = 0$

11/11/93 The Archamedian Paper Trisector

If one accepts the facts of the original figure, one only need prove that BK = AB.
If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?



$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

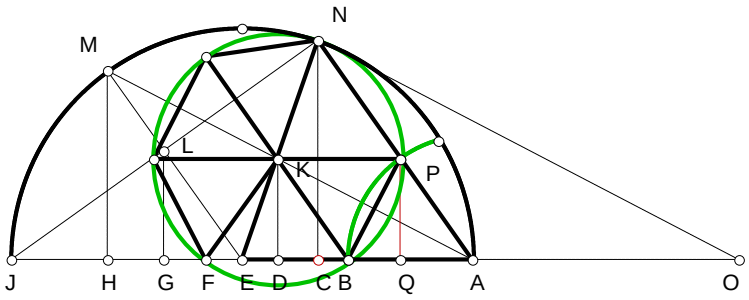
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

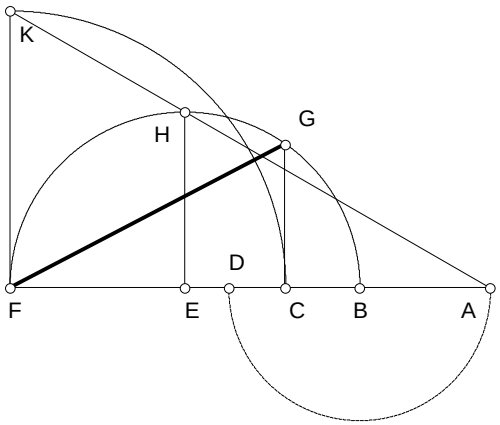
$AB - BK = 0$ $AB = 0.25$

If PK is parallel to AJ, then . . . .



$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

11/12/93 To Square A Circle Off The Base Of A Right Triangle.



Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?

Using the approximation,π = 22/7, square the circle off the base of a right triangle.

$BF := 1$ $BE := \frac{BF}{2}$ $EF := BE$ $EH := BE$ $BD := \frac{3}{4} \cdot BE$ $AB := BD$

$AE := AB + BE$ $AF := AB + BF$ $FK := \frac{EH \cdot AF}{AE}$ $CF := FK$ $BC := BF - CF$

$CG := \sqrt{BC \cdot CF}$ $FG := \sqrt{CF^2 + CG^2}$ $\pi_A := \frac{FG^2}{BE^2}$

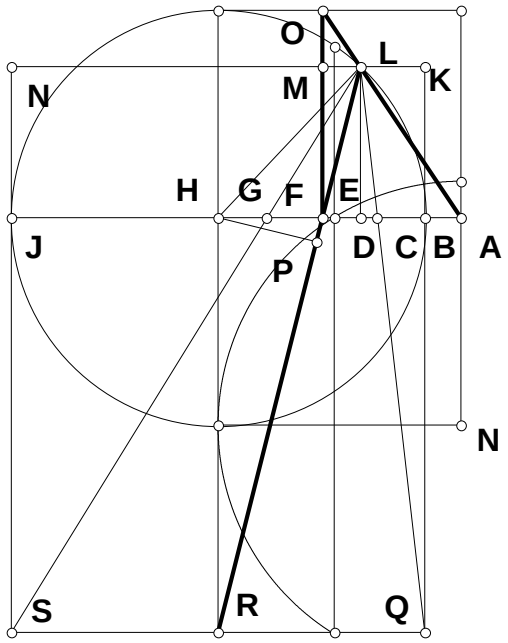
$\pi = 3.14159265359$

$\pi_A = 3.142857142857$

$\frac{\pi}{\pi_A} = 0.999597662505843$

11/18/93 Exploring Cube Roots Plate A

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.



$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM$

$LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

$BQ := BJ \quad BK := DL \quad BD := BF - DF$

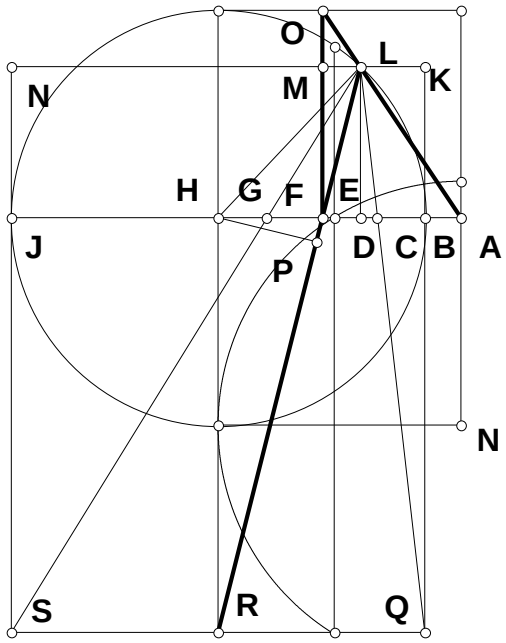
$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad HJ := BH \quad DJ := HJ + FH + DF \quad LN := DJ$

$JS := BJ \quad JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC$

$AG := AB + BG \quad AJ := AB + BJ \quad \left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

11/18/93 Exploring Cube Roots Plate A

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.



$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM$

$LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

$BQ := BJ \quad BK := DL \quad BD := BF - DF$

$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad DJ := BJ - BD \quad LN := DJ \quad JS := BJ$

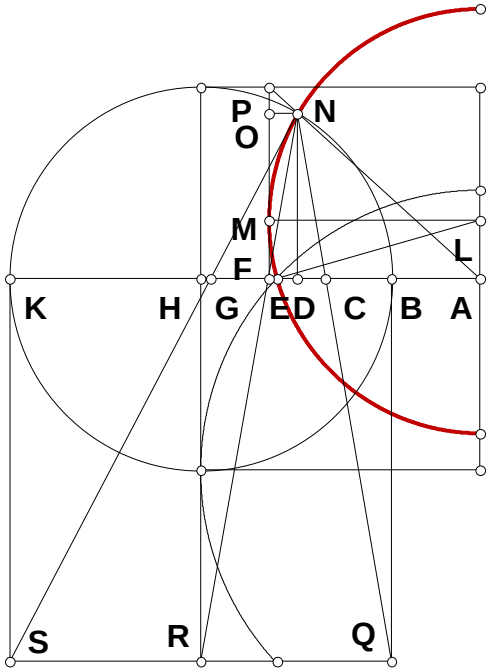
$JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC \quad AG := AB + BG$

$AJ := AB + BJ$

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

11/18/93 Exploring Cube Roots Plate B

If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.



$N := 1.2$ $BK := 1$

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

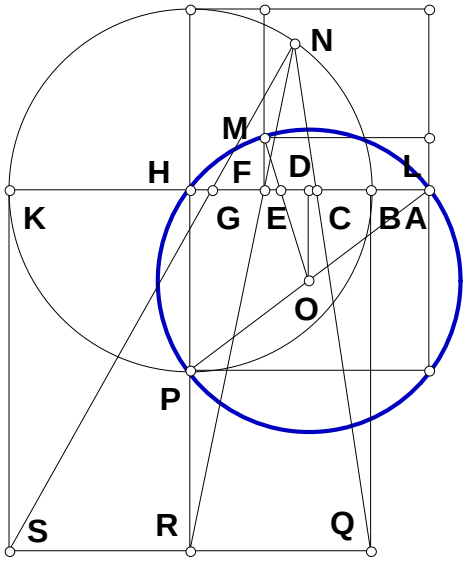
$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$ $AB = 1.523$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

11/18/93 Exploring Cube Roots Plate C

The circle AO passes through point M.



$N := 2$

$AB := 1 \quad AK := AB \cdot N \quad BK := AK - AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

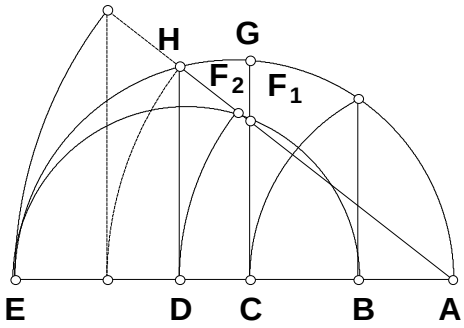
$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$



11/22/93 Cube by Iteration

When F_1 and F_2 are the same point on C, then a sixth root series has been constructed. Use iteration to place F_2 on F_1.

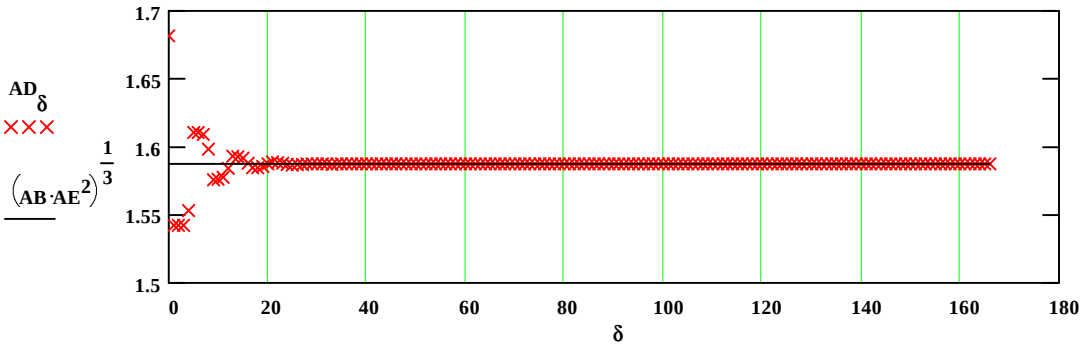
$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[\dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG}\right]^2} \end{bmatrix}$$

$$\begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$

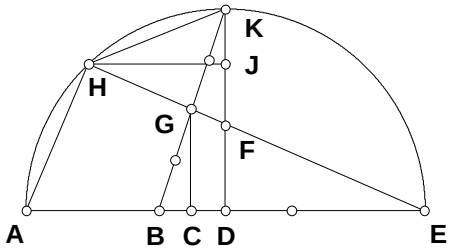
$\Delta \equiv 166$



$$(AB^2 \cdot AE^4)^{\frac{1}{6}} - AD_\Delta = 0 \qquad (AB \cdot AE^2)^{\frac{1}{3}} - AD_\Delta = 0$$

11/24/93 POR Series IV

Generalize the work of 07/25/93 for dividing the base AE with K constant.



$N_1 := 3 \qquad N_2 := 5 \qquad AE := 1$

$\alpha := 1 .. N_1 - 1 \qquad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\frac{AH}{HK} = 0.265 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

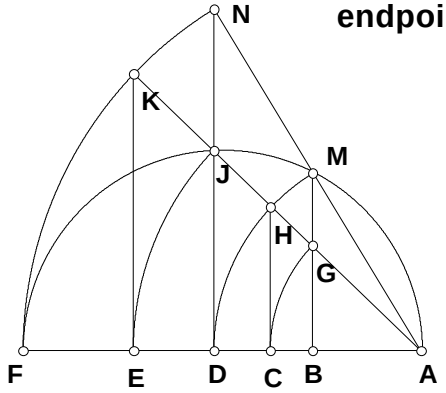
$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$

12/04/93 Exponential Series M^(1/2^N)

Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.



$M := 8$

$AF := M \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

$AN := AF \quad AD := AM \qquad DF := AF - AD$

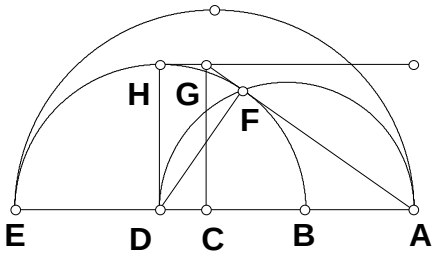
$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

$AK := AF \quad AE := AJ \qquad AH := AD \qquad AC := \frac{AD \cdot AH}{AJ}$

$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$M^{\frac{1}{4}} - AC = 0 \qquad M^{\frac{2}{4}} - AD = 0 \qquad M^{\frac{3}{4}} - AE = 0$

12/06/93 Alternate Method: Square Root
Common Segment Common Endpoint



$N := 6 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2}$

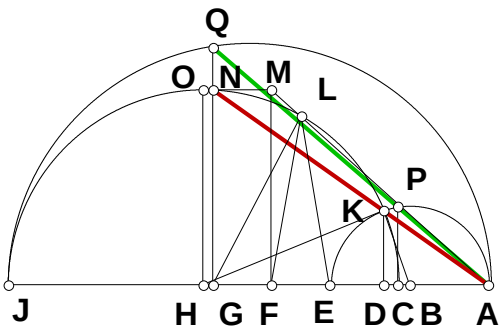
$DF := BD \qquad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$

12/06/93B Gruntwork IV on the Delian Solution

Are A,P and Q collinear? Are A, K and N collinear?



$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$AE := \left(AC^2 \cdot AJ\right)^{\left(\frac{1}{3}\right)} \qquad AG := \left(AC \cdot AJ^2\right)^{\left(\frac{1}{3}\right)}$

$CG := AG - AC \qquad CJ := AJ - AC$

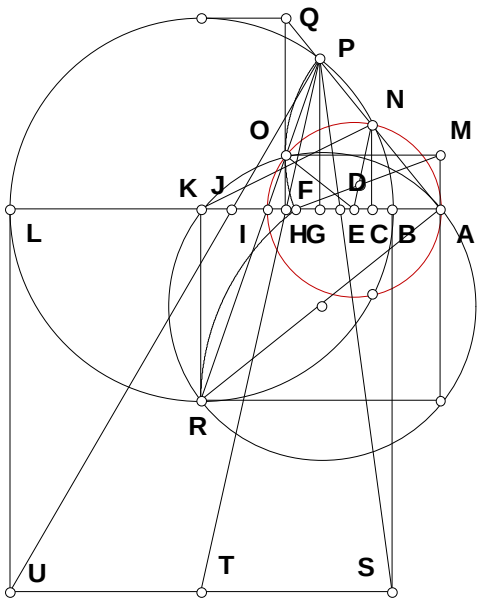
$GJ := CJ - CG \qquad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \qquad CE := AE - AC \qquad CH := \frac{CJ}{2}$

$BK := AB \qquad HK := CH \qquad HJ := CH \qquad AH := AJ - HJ \qquad BH := AH - AB \qquad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$AD := AB + BD \qquad DE := AE - AD \qquad DK := \sqrt{AD \cdot DE} \qquad GQ := \sqrt{AG \cdot GJ} \qquad CP := \sqrt{AC \cdot CE}$

$$\frac{AG}{GN} - \frac{AD}{DK} = 0 \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$$



12/11/93

The structure in red appears to be a constant.

$N := 6$ $AB := 1$ $AL := AB \cdot N$

$BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$

$AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$ $BE := AE - AB$ $BJ := AJ - AB$

$JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$

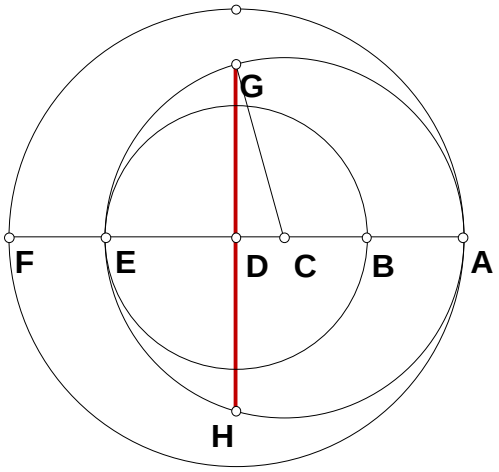
$KR := BK$ $KL := BK$ $FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$

$AD := \frac{AI}{2}$ $KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

12/12/93 The Square Root



Square root by common segment common midpoint. Given AFand BE is GH their root?

$N := 5$ $BE := 1$ $AF := BE \cdot N$

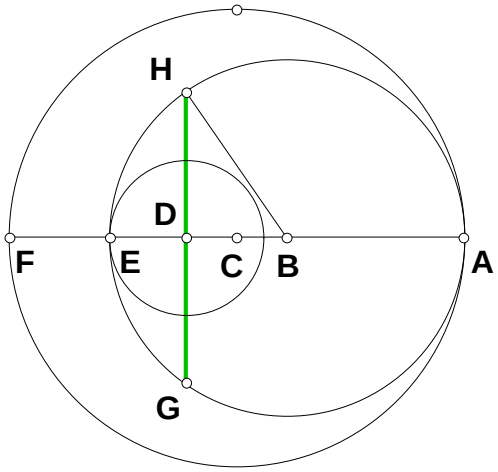
$AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$ $AB := AD - BD$

$AE := BE + AB$ $AC := \frac{AE}{2}$ $CG := AC$

$CD := AD - AC$ $GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$

12/12/93 Generalize The Previous Square Root Figure



$N_1 := 1 \qquad N_2 := 3 \qquad N_3 := 2$

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

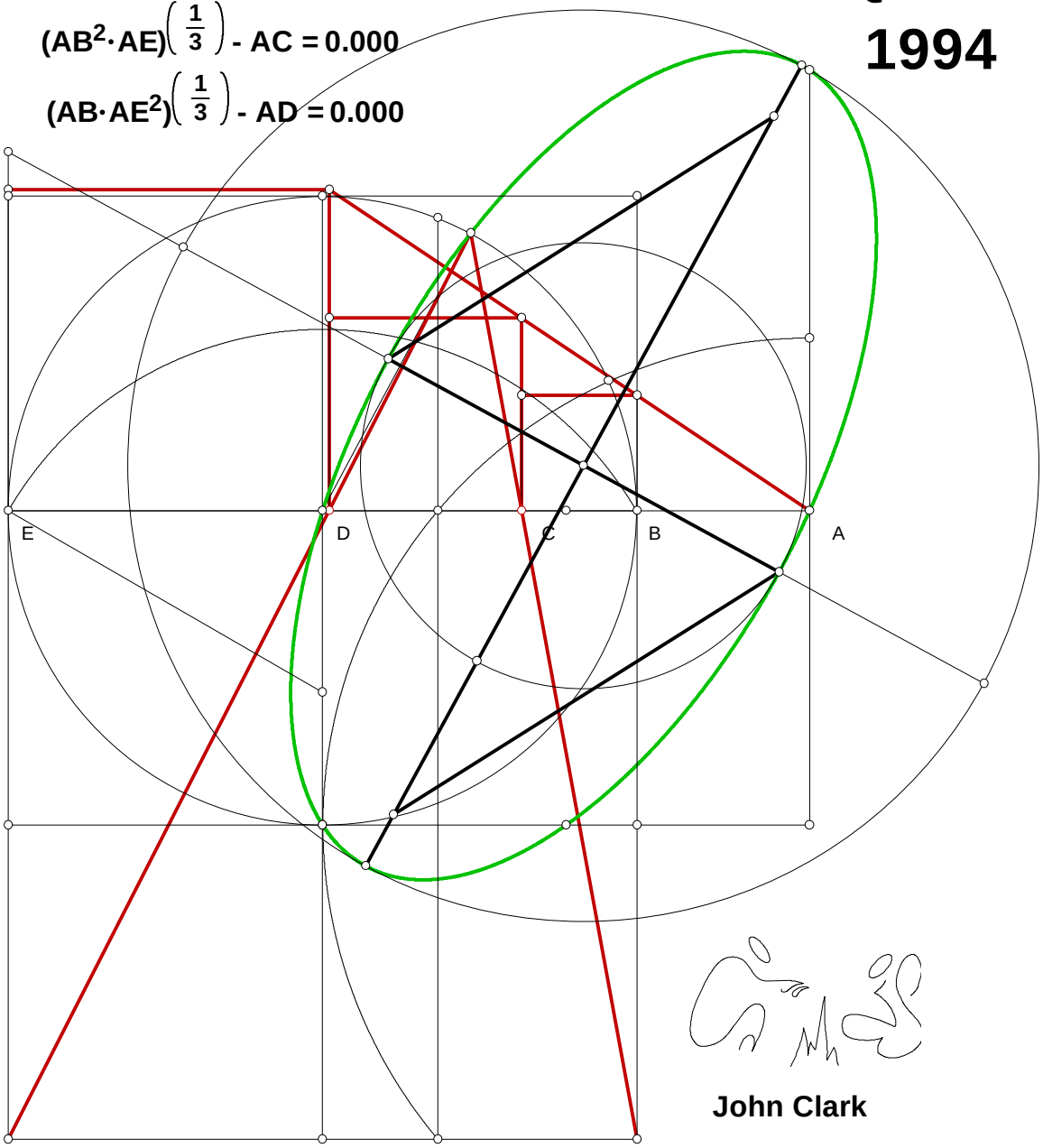
$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

The Delian Quest

1994

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

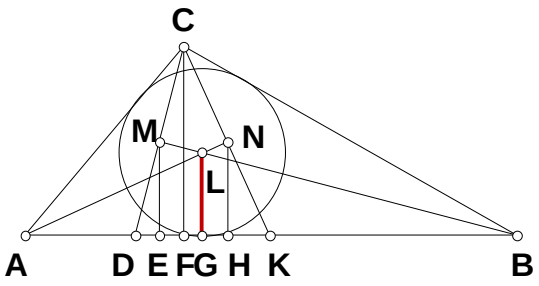
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

04/06/94 Inscribing A Circle In A Given Triangle

Given three sides of a triangle, what is the length of the inscribed radius?



$AB := 3 \qquad BC := 4 \qquad AC := 5$

$AK := AC \qquad BD := BC \qquad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$

$FK := AK - AF \qquad CF := \sqrt{AC^2 - AF^2}$

$CK := \sqrt{FK^2 + CF^2} \qquad AN := \frac{CF \cdot AK}{CK}$

$AH := \frac{CF \cdot AN}{CK} \qquad HN := \frac{FK \cdot AN}{CK} \qquad BF := AB - AF$

$DF := BD - BF \qquad CD := \sqrt{CF^2 + DF^2} \qquad BM := \frac{CF \cdot BD}{CD} \qquad BE := \frac{CF \cdot BM}{CD} \qquad GL := \frac{HN \cdot AB}{AH + BE}$

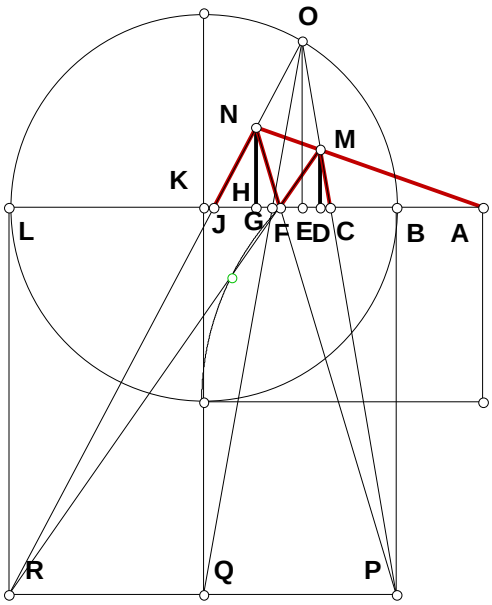
$GL = 1$

$S_1 := \begin{bmatrix} AB \\ BC \\ AC \end{bmatrix} \qquad S_2 := \begin{bmatrix} BC \\ AC \\ AB \end{bmatrix} \qquad S_3 := \begin{bmatrix} AC \\ AB \\ BC \end{bmatrix} \qquad \delta := 0 .. 2$

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

04/21/94 The Cradle

Are A,M,N colinear?



$N := 5 \qquad AB := 1 \qquad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \qquad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

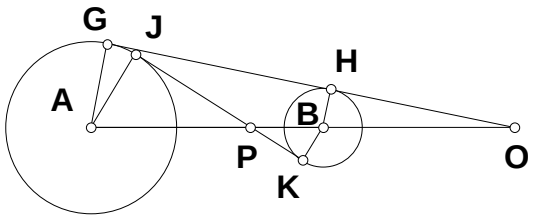
$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

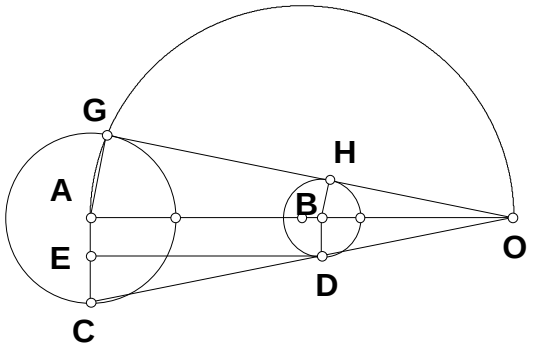
$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$

04/26/94 Tangents and Similarity Points





What is the Algebraic names of the similarity points O and P in relation to the radius of each circle and the difference between their centers?

I will work with point O first.

Given R_L = large radius
R_S = small radius
D = difference between origins.

$R_L := 4 \qquad R_S := 1 \qquad D := 8$

$AC := R_L \qquad BD := R_S \qquad AB := D$

$DE := AB \qquad AE := BD \qquad CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$

EOR_L "External similarity point Origin to center of Radius Large"

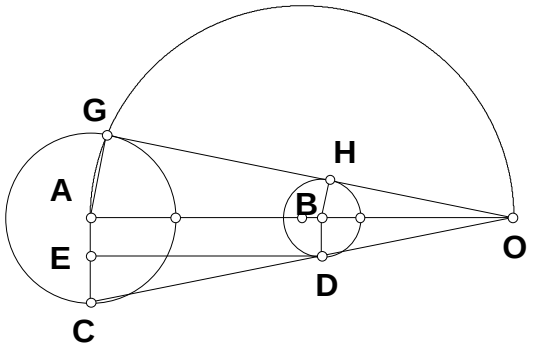
$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right) \qquad EOR_L = 10.667$$

What is the length of line (OG) tangent to both circles?

$AG := AC \qquad GO := \sqrt{AO^2 - AG^2} \qquad GO = 9.888$

And what is the formula?
EOT_{LR} " External similarity point Origin to Tangent (Large Radius)"

$$EOT_{LR} := R_L \cdot \frac{\sqrt{\left(R_L - R_S + D\right) \cdot \left(-R_L + R_S + D\right)}}{R_L - R_S} \qquad EOT_{LR} = 9.888$$



What is the length of the line tangent to the least circle (HO)?

$BH := BD$ $BO := AO - AB$ $BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

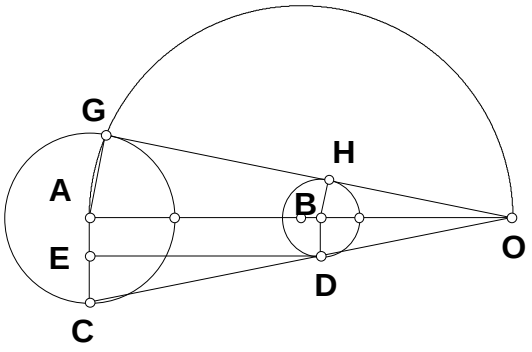
$HO = 2.472$

And what is the formula?

EOT_{SR} " External similarity point Origin to Tangent (Small Radius)"

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S} \qquad EOT_{SR} = 2.472$$

Lastly what is the length of line from tangent to tangent of these circles?



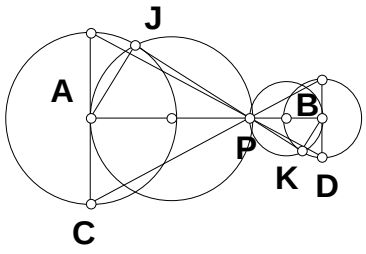
$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

And what is the formula?
ETT "Tangent to Tangent"

$$ETT := \sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}$$

$ETT = 7.416$



I will now turn my attention to the point P, the internal similarity point.

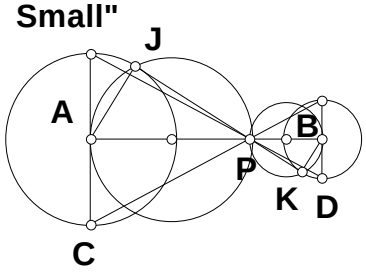
$$AP := \frac{AB \cdot AC}{AC + BD} \qquad AP = 6.4$$

IOR_L "Internal similarity point to center of Radius Large"

$$IOR_L := D \cdot \frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

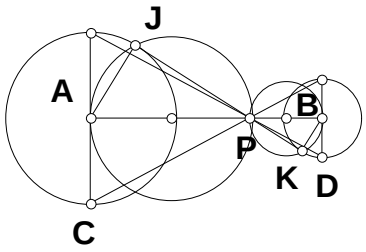
IOR_S "Internal similarity point to center of Radius Small"



$$IOR_S := D \cdot \frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$

$AJ := AC \qquad BK := BD \qquad JP := \sqrt{AP^2 - AJ^2} \qquad JP = 4.996$

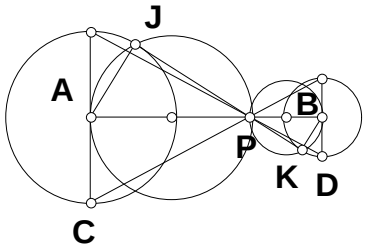
IOT_{LR} "Internal similarity point Origin to Tangent (Large Radius)"



$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D) \cdot (R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996 \qquad KP := \sqrt{BP^2 - BK^2} \qquad KP = 1.249$

IOT_{SR} "Internal similarity point Origin to Tangent (Small Radius)"



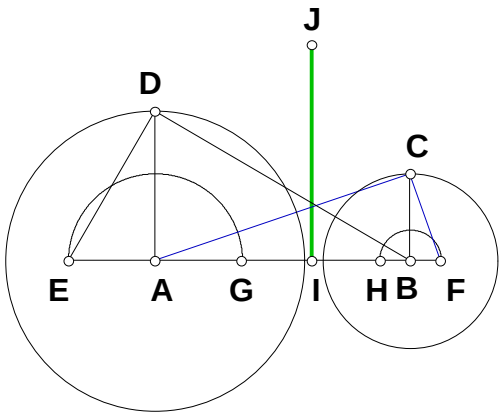
$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D) \cdot (R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249 \qquad JK := JP + KP \qquad JK = 6.245$

ITT "Internal similarity point Tangent to Tangent"

$$ITT := \sqrt{-(R_L + R_S - D) \cdot (R_L + R_S + D)} \qquad ITT = 6.245$$

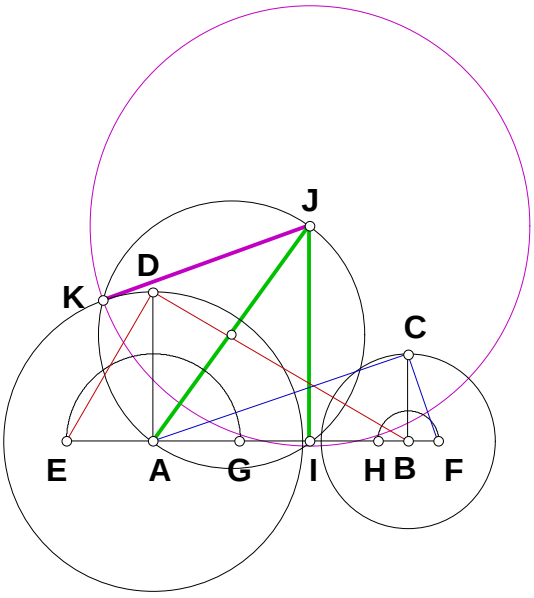
04/27/94 The Chordal or Power Line of two Circles



The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.
One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.
Given two circles find their chordal or power line given just their radius and difference between their centers.

$AB := 1.323 \quad AD := .771 \quad BC := .448 \quad AE := \frac{AD^2}{AB} \quad BF := \frac{BC^2}{AB} \quad GH := AB - (AE + BF)$

$GI := \frac{GH}{2} \quad AI := AE + GI \quad BI := BF + GI$



$D := AB \quad R_1 := AD \quad R_2 := BC$

$$AI - \frac{\left(R_1^2 + D^2 - R_2^2\right)}{2 \cdot D} = 0$$

$$BI - \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D} = 0$$

If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.

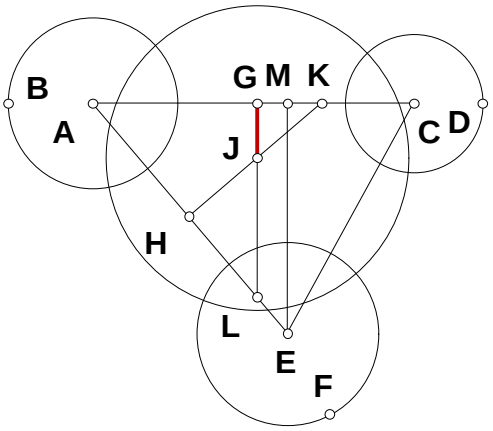
$IJ := 1.112 \quad AJ := \sqrt{AI^2 + IJ^2} \quad AK := AD$

$JK := \sqrt{AJ^2 - AK^2} \quad P := IJ$

$$JK - \frac{\sqrt{R_1^4 - 2 \cdot R_1^2 \cdot D^2 - 2 \cdot R_1^2 \cdot R_2^2 + D^4 - 2 \cdot R_2^2 \cdot D^2 + R_2^4 + 4 \cdot P^2 \cdot D^2}}{2 \cdot D} = 0$$

04/28/94 Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the Algebraic name for the power point and the length of the resultant tangent.



$AB := .438 \quad CD := .354 \quad EF := .471$

$AC := 1.667 \quad AE := 1.559 \quad CE := 1.357$

$R_1 := AB \quad R_2 := CD \quad R_3 := EF$

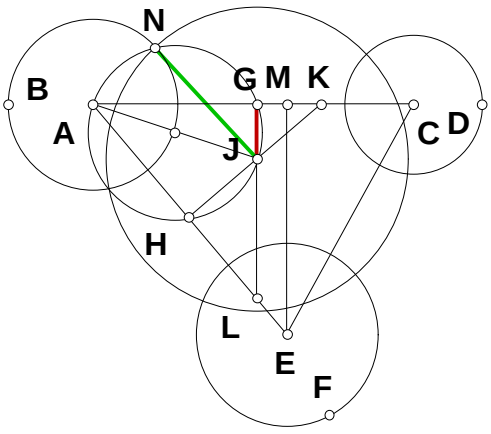
$D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{R_1^2 + D_1^2 - R_2^2}{2 \cdot D_1}$$

$$AH := \frac{R_1^2 + D_2^2 - R_3^2}{2 \cdot D_2}$$

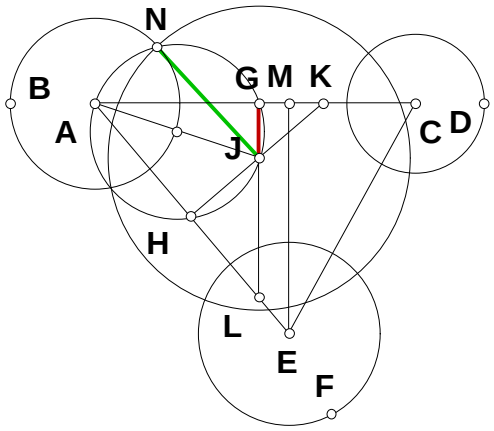
$$AM := \frac{D_2^2 + D_1^2 - D_3^2}{2 \cdot D_1} \qquad EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{1}{2} \cdot \frac{\left[\begin{array}{l}\left(D_2^2 \cdot D_1^2 - D_2^2 \cdot R_1^2 + D_2^2 \cdot R_2^2 + D_3^2 \cdot R_1^2 - 2 \cdot D_1^2 \cdot R_3^2\right) \ldots \\ + R_2^2 \cdot D_1^2 - D_3^2 \cdot R_2^2 - D_1^4 + D_3^2 \cdot D_1^2 + D_1^2 \cdot R_1^2\end{array}\right]}{D_1 \cdot \sqrt{-\left(D_2 + D_1 - D_3\right) \cdot \left(D_2 + D_1 + D_3\right) \cdot \left(D_2 - D_1 - D_3\right) \cdot \left(D_2 - D_1 + D_3\right)}} = 0$$



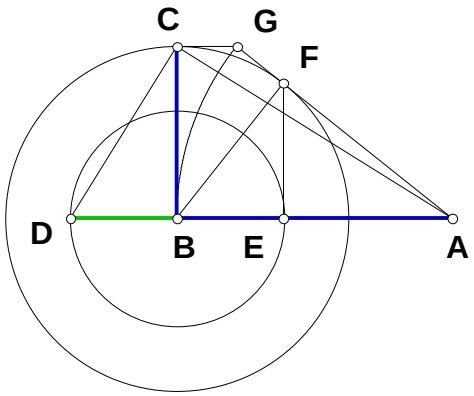
$$AJ := \sqrt{AG^2 + GJ^2} \qquad AN := AB \qquad JN := \sqrt{AJ^2 - AN^2}$$

$JN = 0.786$



$$JN - \sqrt{\frac{-\left[\begin{array}{l}\left(-D_3^2 \cdot R_1^2 \cdot D_1^2 + R_1^2 \cdot D_3^4 + R_2^4 \cdot D_2^2 + D_1^2 \cdot D_3^2 \cdot D_2^2\right) \ldots \\ + \left(R_1^4 \cdot D_3^2 - R_2^2 \cdot D_1^2 \cdot D_2^2 + R_2^2 \cdot D_2^4 + R_1^2 \cdot R_2^2 \cdot D_1^2\right) \ldots \\ + \left(-D_2^2 \cdot D_1^2 \cdot R_3^2 - D_1^2 \cdot R_3^2 \cdot R_1^2 - D_1^2 \cdot R_3^2 \cdot D_3^2 - D_1^2 \cdot R_3^2 \cdot R_2^2\right) \ldots \\ + \left(-D_2^2 \cdot R_1^2 \cdot D_3^2 + R_3^2 \cdot D_3^2 \cdot R_2^2 - R_1^2 \cdot R_2^2 \cdot D_2^2 + D_1^4 \cdot R_3^2\right) \ldots \\ + \left(-R_1^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot R_3^2 - D_3^2 \cdot R_1^2 \cdot R_3^2\right) \ldots \\ + D_2^2 \cdot R_1^2 \cdot R_3^2 + D_1^2 \cdot R_3^4\end{array}\right]}{\left(D_2 - D_3 + D_1\right) \cdot \left(D_2 + D_3 + D_1\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(-D_2 - D_3 + D_1\right)}} = 0$$

04/30/94 Division №



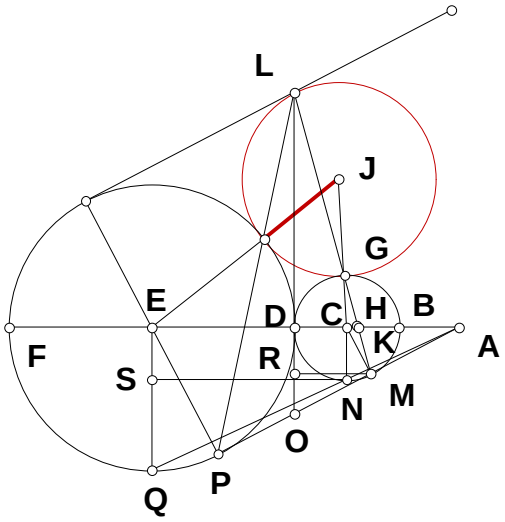
$N_1 := 5$ $N_2 := 2$

$AB := N_1$ $BC := N_2$ $AC := \sqrt{AB^2 + BC^2}$

$CD := \frac{BC \cdot AC}{AB}$ $BD := \sqrt{CD^2 - BC^2}$

$\frac{N_2^2}{N_1} - BD = 0$

05/01/94 Two Circles And A Parallel



Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.

$R_1 := 3 \qquad R_2 := 2$

$DE := R_1 \qquad BC := R_2 \qquad CN := BC$

$EQ := DE \qquad CD := BC \qquad CE := CD + DE$

$ES := CN \qquad NS := CE \qquad SQ := EQ - ES$

$AE := \frac{NS \cdot EQ}{SQ} \qquad AD := AE - DE \qquad EP := DE$

$AP := \sqrt{AE^2 - EP^2} \qquad DO := \frac{EP \cdot AD}{AP}$

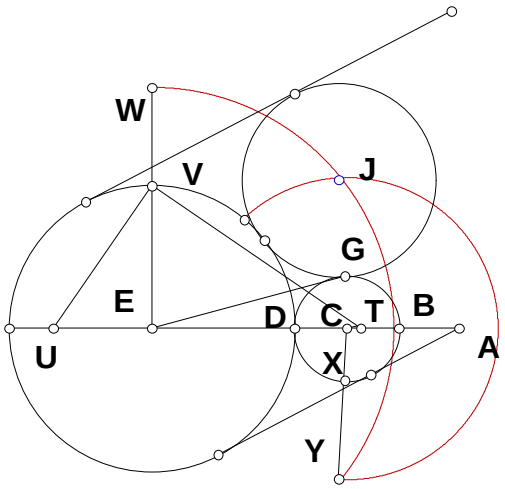
$DL := \frac{DO \cdot DE}{CD} \qquad AC := AD - CD \qquad AM := \frac{AP \cdot AC}{AE} \qquad AO := \frac{AE \cdot AD}{AP} \qquad MO := AO - AM$

$MR := \frac{AD \cdot MO}{AO} \qquad RO := \frac{DO \cdot MR}{AD} \qquad DR := DO - RO \qquad LR := DR + DL \qquad ML := \sqrt{MR^2 + LR^2}$

$DK := \frac{MR \cdot DL}{LR} \qquad CK := DK - CD \qquad CH := \frac{LR \cdot CK}{ML} \qquad CM := BC \qquad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \qquad GL := ML - MG \qquad GJ := \frac{CM \cdot GL}{MG}$

The Algebraic name for GJ suggests a simpler method of construction.



$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad GJ - R_3 = 0$

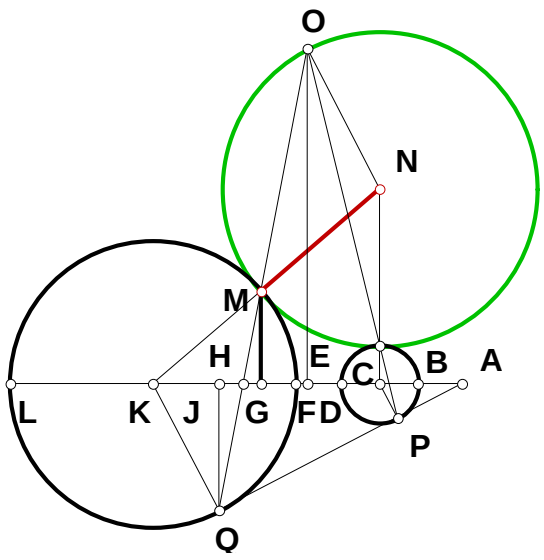
$ET := 4 \cdot BC \qquad EV := DE \qquad EU := \frac{DE^2}{ET}$

$VW := EU \qquad XY := EU \qquad EV := DE \qquad CX := BC$

$EW := EV + VW \qquad CY := CX + XY$

$GJ_2 := EU \qquad GJ - GJ_2 = 0$

05/04/94 Two Circles And A Tangent



Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.

$R_1 := 1 \quad R_2 := .5 \quad D := 2 \quad N := 2$

$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK$

$$AK := \frac{D \cdot R_1}{R_1 - R_2} \qquad EK := \frac{R_1^2 + D^2 - R_2^2}{2 \cdot D}$$

$$AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$$

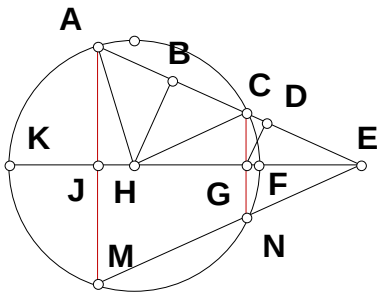
$$FG := \frac{FL}{N} \quad GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK} \quad AF := AK - FK \quad FJ := AJ - AF$$

$$JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG \quad QM := \sqrt{(JQ + GM)^2 + GJ^2} \quad GH := \frac{GJ \cdot GM}{JQ + GM}$$

$$HM := \frac{QM \cdot GM}{JQ + GM} \quad EF := EK - FK \quad EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \quad MO := HO - HM$$

$$KM := FK \quad MN := \frac{KM \cdot MO}{QM} \qquad MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2\,N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

05/06/94 A Ratio In Trisection



What is AJ to CG?

$N := 2.423$ $FH := 1$ $CE := FH$

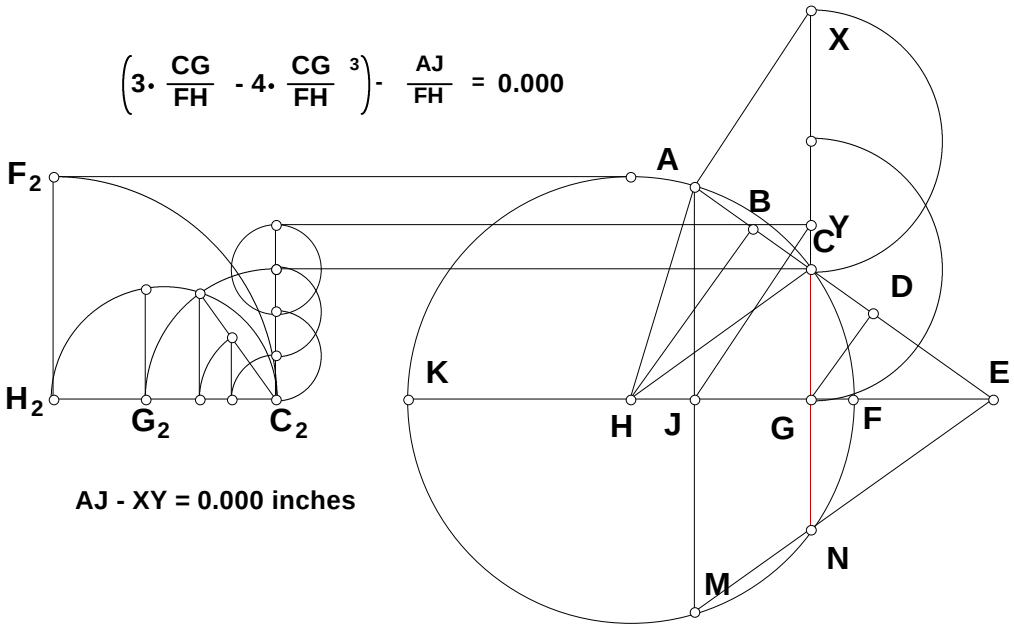
$CG := \frac{FH}{N}$ $EG := \sqrt{CE^2 - CG^2}$ $CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2}$ $EH := 2 \cdot EG$ $BH := \frac{DG \cdot EH}{EG}$ $CH := FH$

$BC := \sqrt{CH^2 - BH^2}$ $AC := 2 \cdot BC$ $AE := AC + CE$ $AJ := \frac{CG \cdot AE}{CE}$ $3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0$

$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$ $AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$ $AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

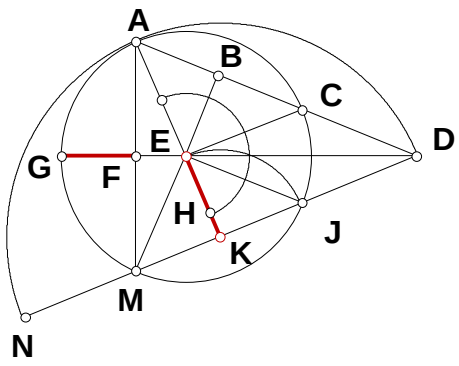
$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG}{FH}^3\right) - \frac{AJ}{FH} = 0.000$



AJ - XY = 0.000 inches

05/07/94 A Trisection Ratio

In trisection, what is the ratio of FG/EK?



$N := 2$

$AE := 1 \quad EH := \frac{AE}{2} \quad HK := AE \cdot N \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

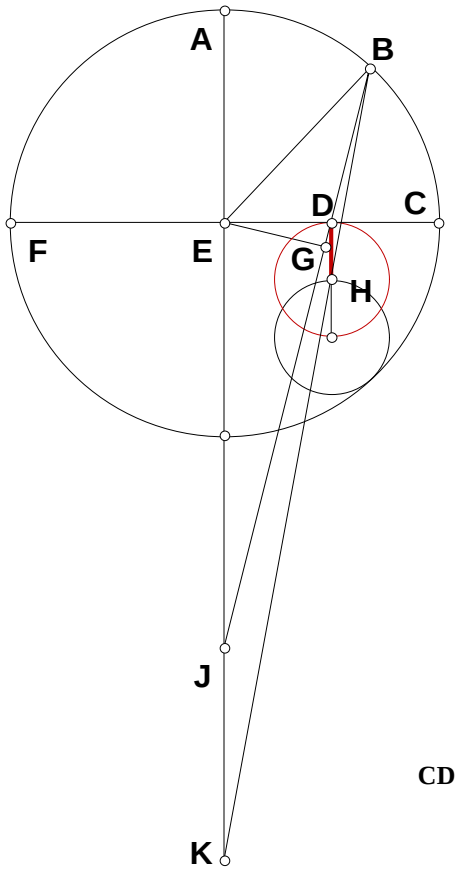
Algebraic Names,

$$FG - \left[1 + \frac{(N+1)\cdot(2\cdot N-1)}{(2N+1)\cdot\sqrt{(N+1)\cdot(2N+1)}}\right] = 0 \qquad EK - \frac{1}{2} - N = 0$$

$$\frac{FG}{EK} - 2\cdot\frac{\left[2\cdot N\cdot\sqrt{(N+1)\cdot(2\cdot N+1)} + \sqrt{(N+1)\cdot(2\cdot N+1)} + (N+1)\cdot(2\cdot N-1)\right]}{\left[(2\cdot N+1)^2\cdot\sqrt{(N+1)\cdot(2\cdot N+1)}\right]} = 0$$

05/16/94A Tangent Diameter and Circles

Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.



$CF := 1 \qquad CE := \frac{CF}{2} \qquad N_1 := 2 \qquad N_2 := 3$

$CD := \frac{CE}{N_1} \qquad DE := CE - CD \qquad EJ := CE \cdot N_2$

$DJ := \sqrt{DE^2 + EJ^2} \qquad JG := \frac{EJ^2}{DJ} \qquad BE := CE$

$EG := \sqrt{EJ^2 - JG^2} \qquad BG := \sqrt{BE^2 - EG^2} \qquad BJ := BG + JG$

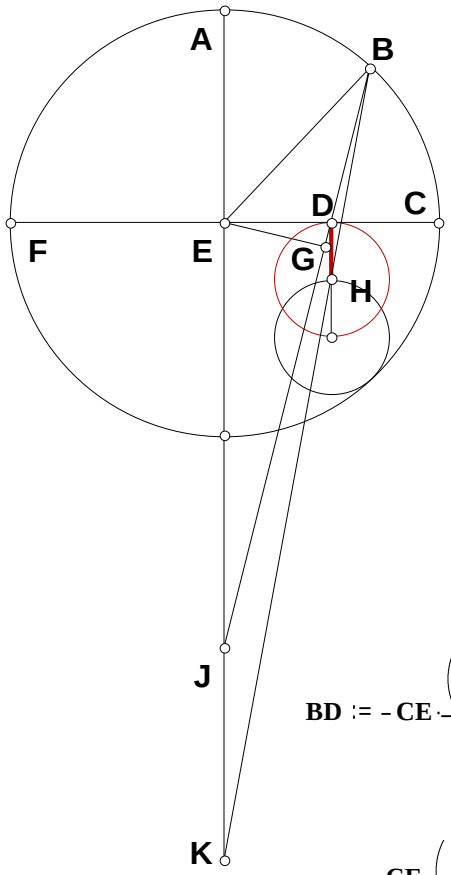
$JK := CE \qquad BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$

Algebraic Names:

$$CD := \frac{CE}{N_1} \qquad DE := CE \cdot \frac{(N_1 - 1)}{N_1} \qquad EJ := CE \cdot N_2$$

$$DJ := CE \cdot \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{N_1}$$

$$JG := CE \cdot \frac{N_1 \cdot N_2^2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} \qquad EG := CE \cdot \frac{(N_1 - 1) \cdot N_2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$



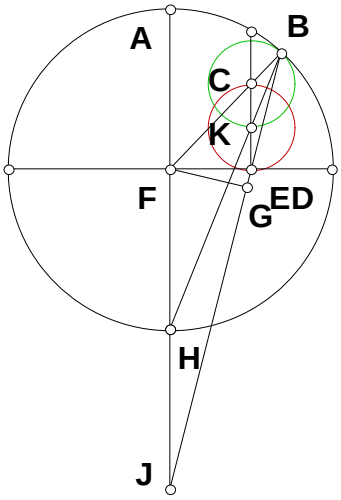
$$BG := CE \cdot \sqrt{\frac{\left(N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2\right)}{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}}$$

$$BJ := CE \cdot \frac{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

$$BD := -CE \cdot \frac{\left(-N_1 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_1^2 - 2 \cdot N_1 + 1\right)}{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1\right)}$$

$$DH := \frac{CE \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1\right)}{N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}$$

05/16/94B Tangent Diameter and Circles



Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.

$N_1 := 2 \quad N_2 := 2 \quad AF := 1$

$DF := AF \quad DE := \frac{DF}{N_1} \quad AJ := AF \cdot N_2$

$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$

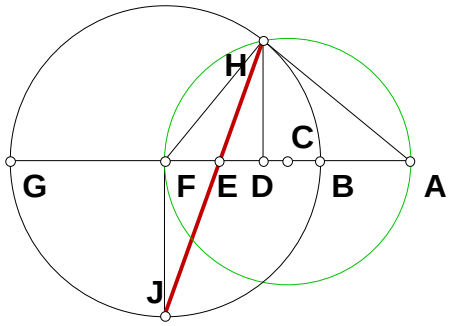
$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad KE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot KE\right]^2} \quad BC - KE = 0 \quad KE = 0.375$

$$1 - \frac{2 \cdot N_1^2 - 2 \cdot N_1 + 1 + N_1^2 \cdot N_2^2 - 2 \cdot N_1^2 \cdot N_2}{N_1 \cdot \left(\sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 4 \cdot N_1 \cdot N_2 - N_2^2 + 2 \cdot N_2} + N_1 + N_1 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2\right)} - BC = 0$$

10/27/94 Trivial Method Square Root

AE is the square root of AB x AG.



$N := 5$ $AB := 1$

$AG := AB \cdot N$ $BG := AG - AB$ $BF := \frac{BG}{2}$

$AF := AB + BF$ $FH := BF$ $DF := \frac{FH^2}{AF}$ $AD := AF - DF$

$BD := AD - AB$ $DG := BG - BD$ $FJ := BF$

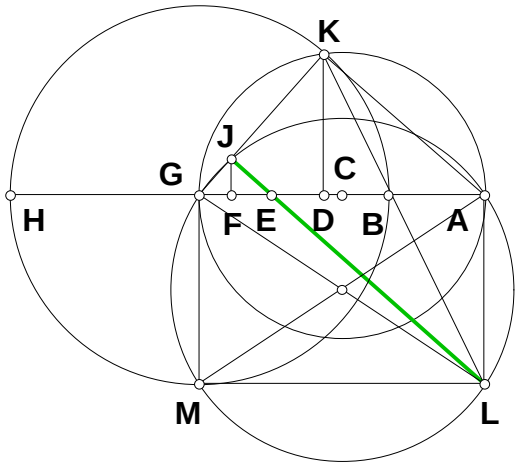
$DH := \sqrt{BD \cdot DG}$ $DE := \frac{DF \cdot DH}{DH + FJ}$ $AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$

10/28/94 Trivial Method Square Root

AE is the square root of AB x AH.

$N := 5 \quad AB := 1 \quad AH := AB \cdot N \quad BH := AH - AB$



$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

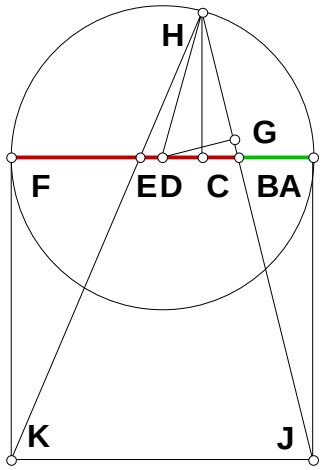
$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad JL := \sqrt{GL^2 - GJ^2}$

$FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$

10/31/94 Square Root of a Segment

Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.



$N := 11 \qquad AB := 1$

$AF := AB \cdot N \qquad BF := AF - AB \qquad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \qquad BC := \frac{AB \cdot (BG + GH)}{BJ} \qquad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \qquad CE := \frac{CF \cdot CH}{(CH + FK)}$

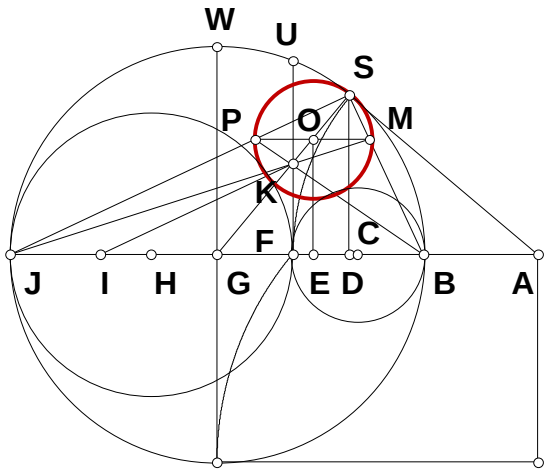
$EF := CF - CE \qquad BE := BC + CE \qquad DF := BF - BE \qquad CD := AD - AC \qquad BE^2 - EF = 0$

$$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$$

$$EF - \left(N - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} - 1 \right) = 0$$

12/24/94 Power Line At Square Root

In this square root figure, what is the Algebraic name of the tangent circle OS?



$N := 5 \qquad AB := 1 \qquad AJ := AB \cdot N$

$AF := \sqrt{AB \cdot AJ} \quad BJ := AJ - AB \quad BG := \frac{BJ}{2}$

$AG := AB + BG \quad GS := BG \quad DG := \frac{GS^2}{AG}$

$FG := AG - AF \quad BD := BG - DG \quad DJ := BJ - BD$

$DS := \sqrt{BD \cdot DJ} \quad FK := \frac{DS \cdot FG}{DG} \quad BF := AF - AB$

$BK := \sqrt{BF^2 + FK^2} \quad FI := \frac{DJ \cdot FK}{DS} \quad BI := FI + BF$

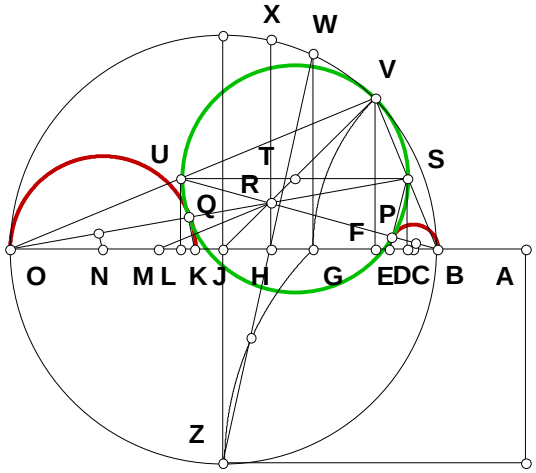
$BP := \frac{BK \cdot BJ}{BI} \quad KP := BP - BK \quad MP := \frac{BJ \cdot KP}{BK} \quad OS := \frac{MP}{2}$

Algebraic Names

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0$$

$$OS - \frac{-2 \cdot N^2 + 2 \cdot N - \sqrt{N} + N^{\left(\frac{5}{2}\right)}}{2 \cdot \left[N^2 - \sqrt{N} - N^{\left(\frac{3}{2}\right)} + 1\right]} = 0$$

12/25/95 Two Prime Exponential Series Developed Through The Powerline Progression



$\Delta := 2$ $\quad \delta := 1 .. \Delta$

$N := 5$ $\quad AB := 1$

$AO := AB \cdot N$ $\quad AG := \sqrt{AB \cdot AO}$ $\quad BO := AO - AB$

$BJ := \frac{BO}{2}$ $\quad JZ := BJ$ $\quad JV := BJ$ $\quad JO := BJ$

$BG_1 := AG - AB$ $\quad GO_1 := BO - BG_1$ $\quad GW_1 := \sqrt{BG_1 \cdot GO_1}$

$GJ_1 := BJ - BG_1$ $\quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{bmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{bmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - \left(BG_\delta + GH_\delta\right) \\ \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]} \\ BJ - \left(BG_\delta + GH_\delta\right) \\ \frac{\left[BJ - \left(BG_\delta + GH_\delta\right)\right] \cdot \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}}{JZ + \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}} \end{bmatrix}$$

$HJ := BJ - BG_\Delta$

$FJ := \frac{(N-1)^2}{2 \cdot (N+1)}$

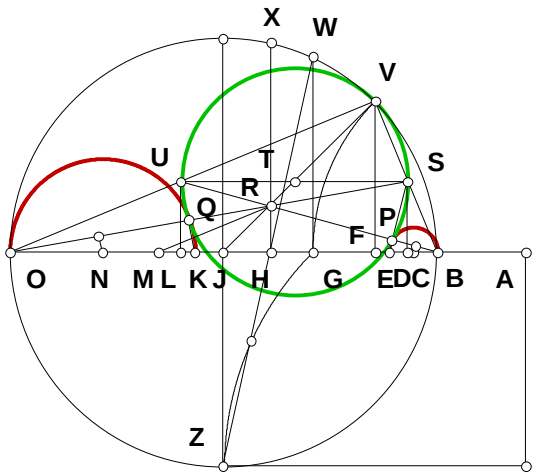
$BF := BJ - FJ$ $\quad FO := FJ + JO$ $\quad FV := \sqrt{BF \cdot FO}$

$HR := \frac{FV \cdot HJ}{FJ}$ $\quad BH := BJ - HJ$ $\quad BR := \sqrt{HR^2 + BH^2}$

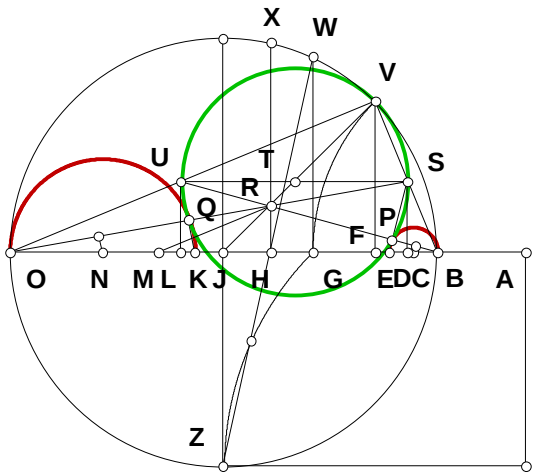
$HM := \frac{FO \cdot HR}{FV}$ $\quad BU := \frac{BR \cdot BO}{BH + HM}$ $\quad RU := BU - BR$

$SU := \frac{BO \cdot RU}{BR}$ $\quad TV := \frac{SU}{2}$ $\quad PU := \frac{BH \cdot SU}{BR}$

$BP := BU - PU$ $\quad BE := \frac{BR \cdot BP}{BH}$ $\quad AE := AB + BE$



$$N^{2^{\frac{1}{\Delta}}} - AE = 0$$



$$LU := \frac{HR \cdot BU}{BR} \qquad BL := \frac{BH \cdot BU}{BR} \qquad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \qquad DS := LU \qquad OS := \frac{OR \cdot DS}{HR}$$

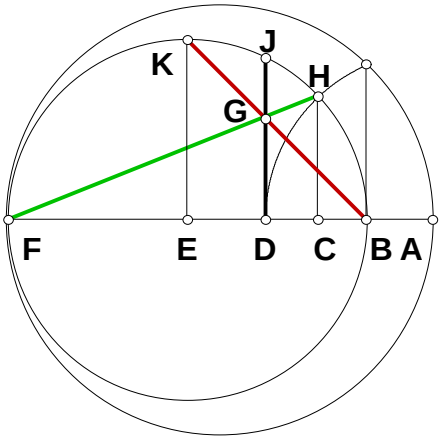
$$DO := \frac{HO \cdot DS}{HR} \qquad QS := \frac{DO \cdot SU}{OS} \qquad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \qquad AK := AO - KO$$

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

12/26/94 Is Point G on DJ?

Is G, the intersection of FH and BK, on DJ?



$N := 5$ $AB := 1$ $AF := AB \cdot N$ $BF := AF - AB$

$BE := \frac{BF}{2}$ $EF := BE$ $EK := BE$

$AD := \sqrt{AB \cdot AF}$ $CE := \frac{(N-1)^2}{2 \cdot (N+1)}$ $CF := CE + EF$

$DF := AF - AD$ $BC := BF - CF$ $CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF}$

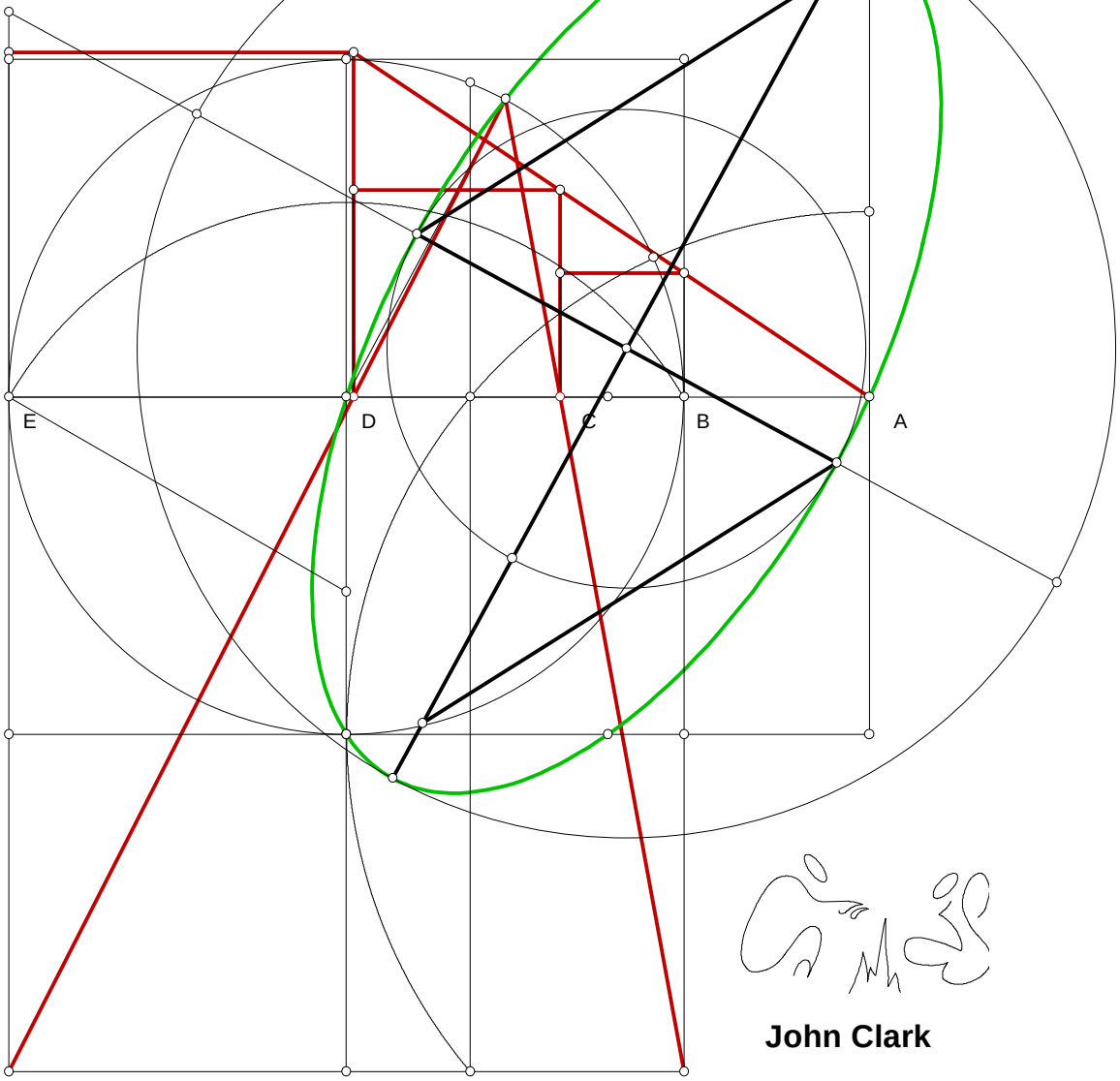
$BD := AD - AB$ $DG_2 := \frac{EK \cdot BD}{BE}$ $DG_1 - DG_2 = 0$

The Delian Quest

1995

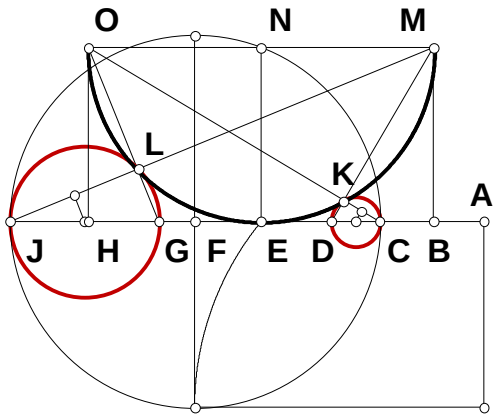
$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

01/06/95 Alternate Method Quad Roots



$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$CJ := AJ - AC \qquad AE := \sqrt{AC \cdot AJ} \qquad CE := AE - AC$

$EJ := CJ - CE \qquad EN := \sqrt{CE \cdot EJ} \qquad BM := EN \qquad HO := EN$

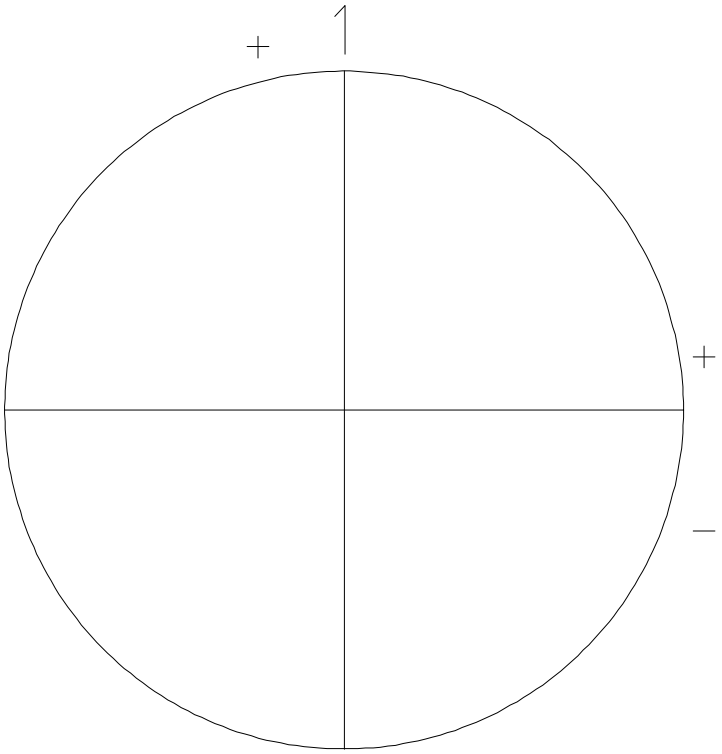
$MN := EN \qquad NO := EN \qquad BE := EN \qquad EH := EN$

$BJ := BE + EJ \qquad MJ := \sqrt{BJ^2 + BM^2} \qquad MO := MN + NO$

$ML := \frac{BJ \cdot MO}{MJ} \qquad JL := MJ - ML \qquad GJ := \frac{MJ \cdot JL}{BJ} \qquad AG := AJ - GJ \qquad \left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH \qquad CO := \sqrt{CH^2 + HO^2} \qquad KO := \frac{CH \cdot MO}{CO} \qquad CK := CO - KO \qquad CD := \frac{CO \cdot CK}{CH} \qquad AD := AC + CD$

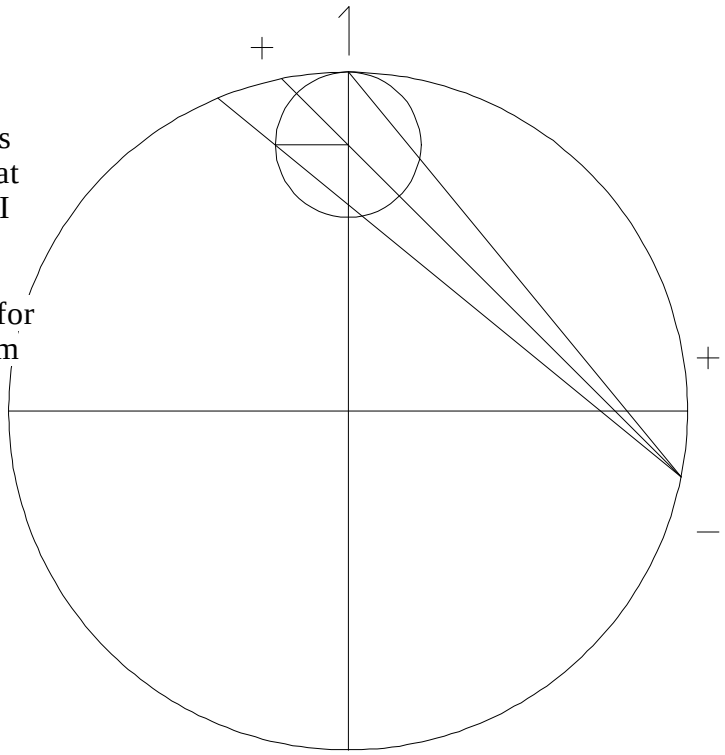
$\left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AG = 0 \qquad N^{\frac{1}{4}} - AD = 0$



Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

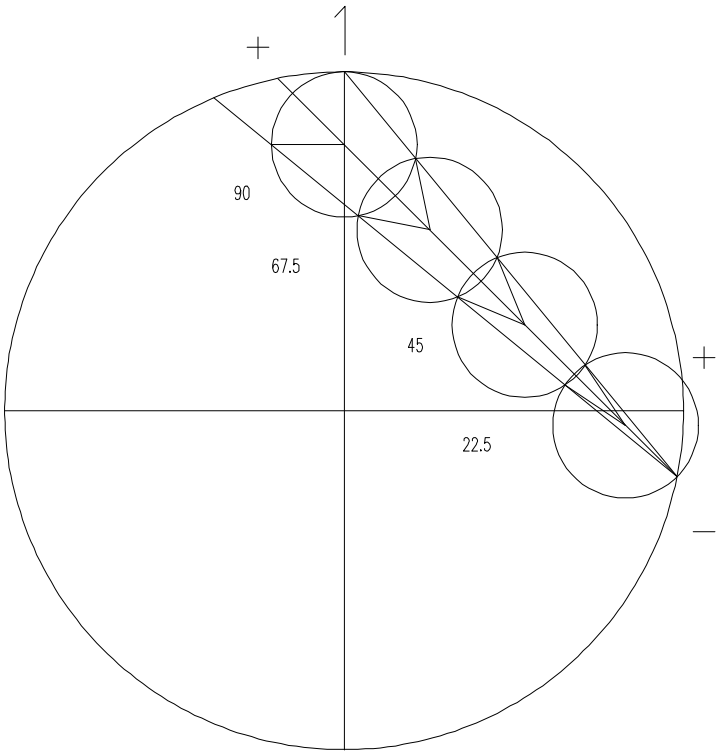
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$



$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

$8 + 1 - 1 = 8$	$8 \cdot 11.25 = 90$
$8 + 1 - 1 - 2 = 6$	$6 \cdot 11.25 = 67.5$
$8 + 1 - 1 - 2 - 2 = 4$	$4 \cdot 11.25 = 45$
$8 + 1 - 1 - 2 - 2 - 2 = 2$	$2 \cdot 11.25 = 22.5$
$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$	

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

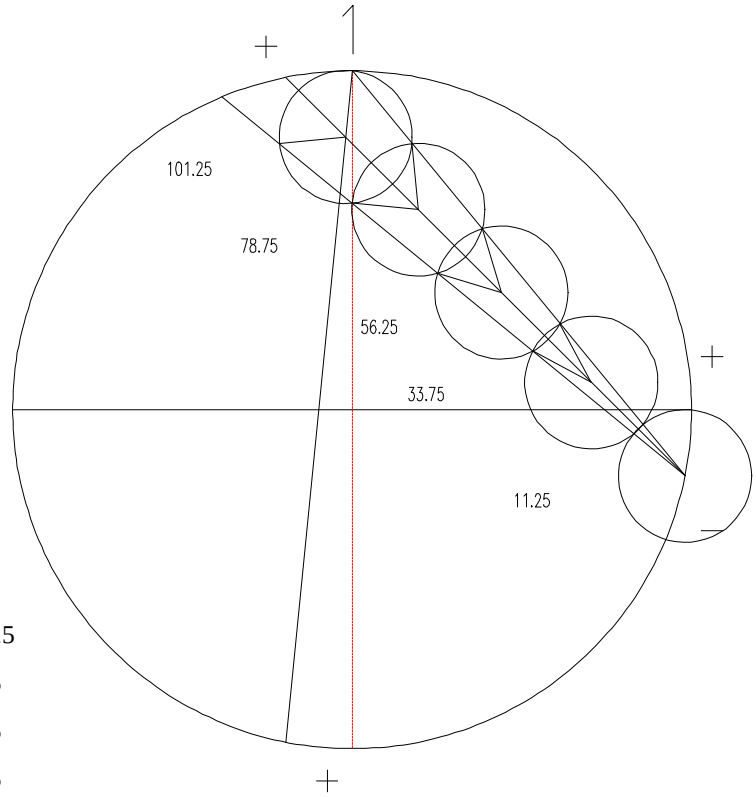
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

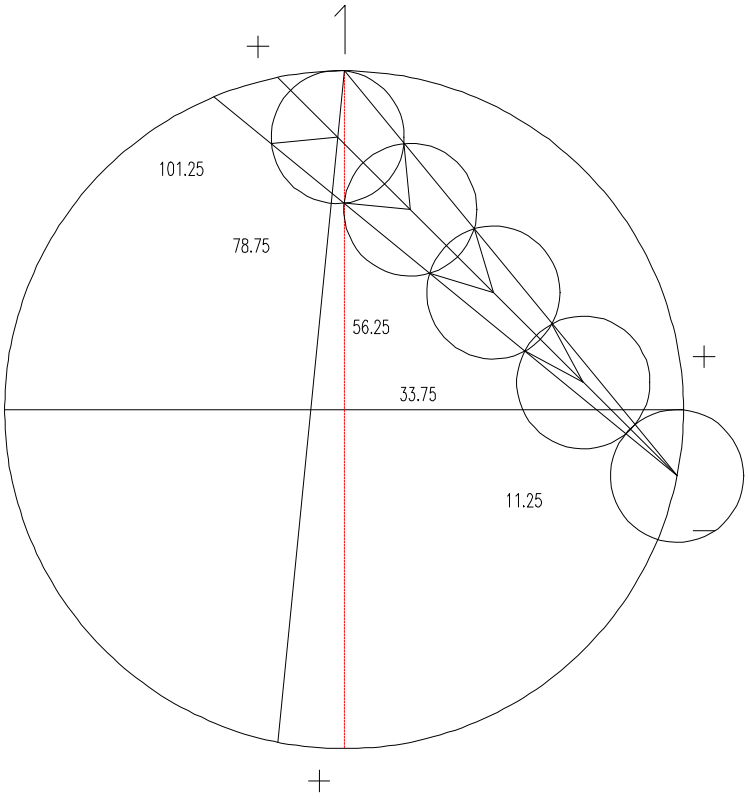
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$



$8 + 1 + 1 - 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 + 1 - 1 - 2 = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 + 1 - 1 - 2 - 2 = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

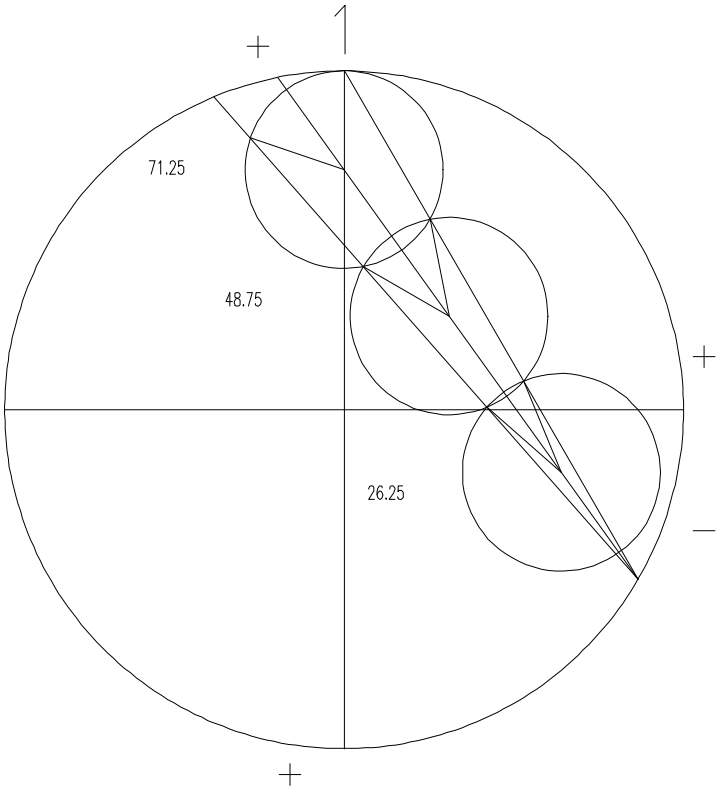
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 - (1 \cdot 2) = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 - (2 \cdot 2) = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 - (3 \cdot 2) = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 - (4 \cdot 2) = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1, 2) = 1$



$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

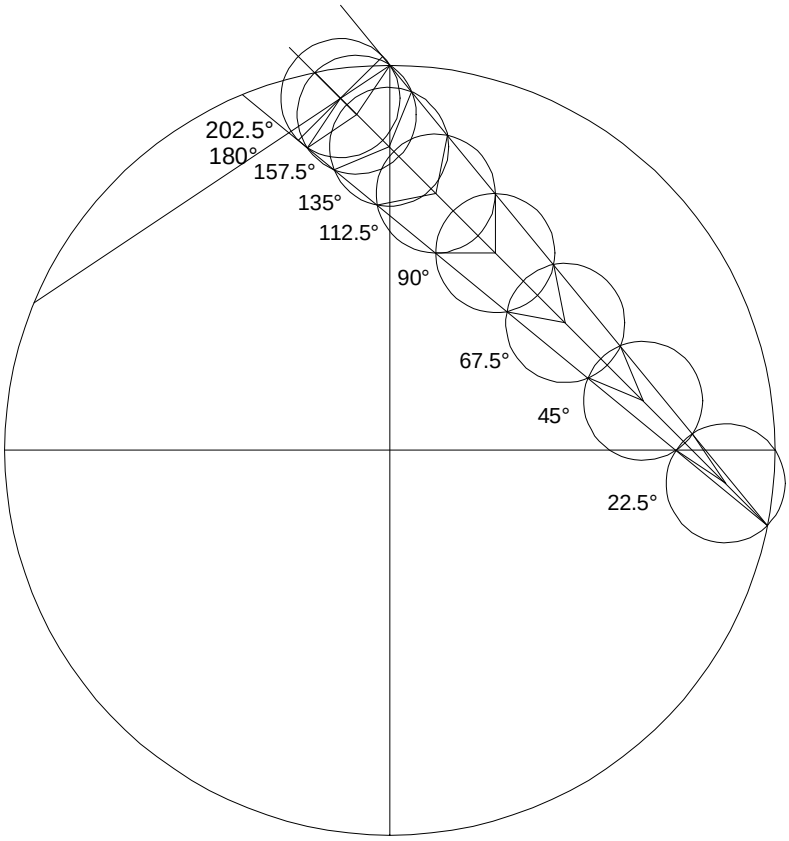
$(24 + 3) - 8 = 19$	$19 \cdot 3.75 = 71.25$
$(24 + 3) - 8 - (1 \cdot 6) = 13$	$13 \cdot 3.75 = 48.75$
$(24 + 3) - 8 - (2 \cdot 6) = 7$	$7 \cdot 3.75 = 26.25$
$(24 + 3) - 8 - (3 \cdot 6) = 1$	$1 \cdot 3.75 = 3.75$

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$



$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

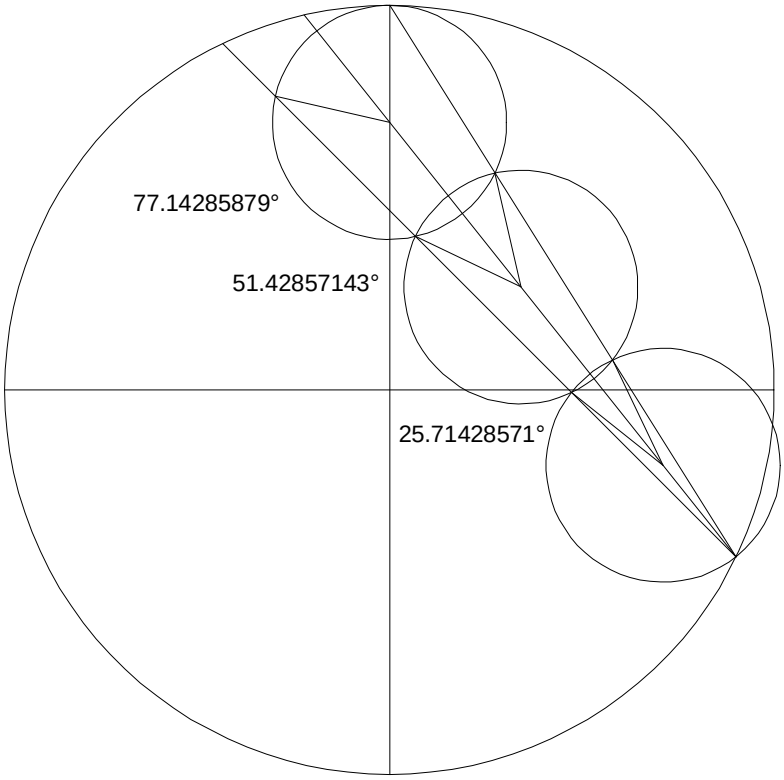
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

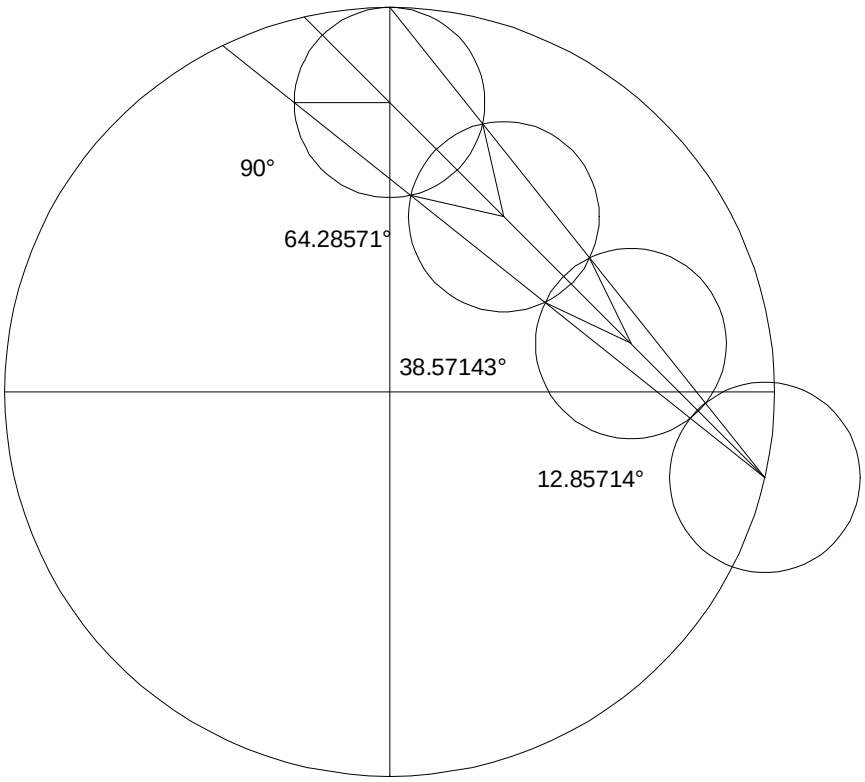
$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$



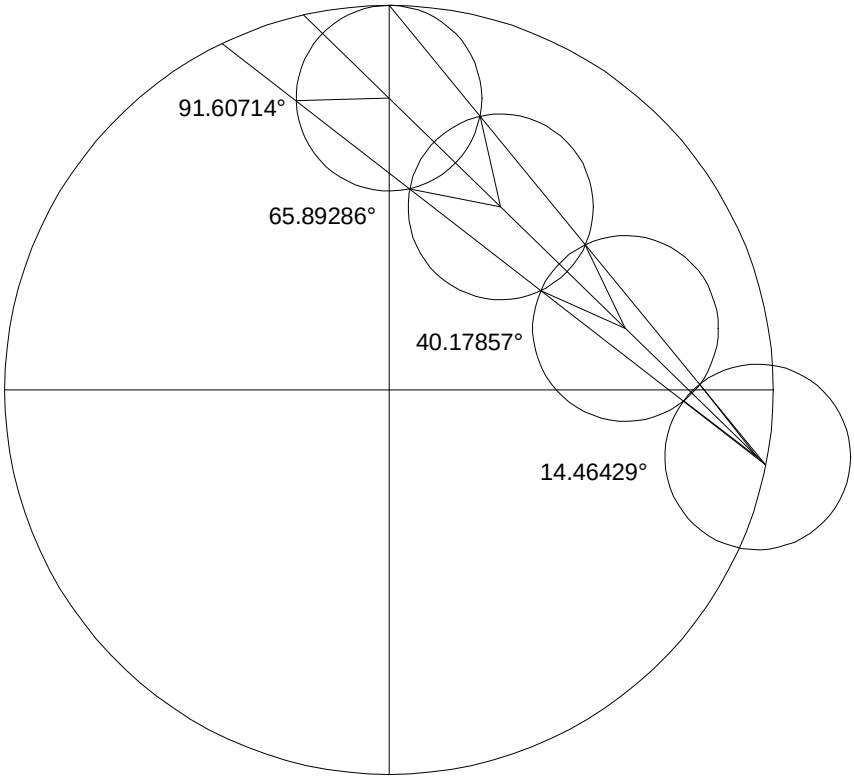
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$



$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

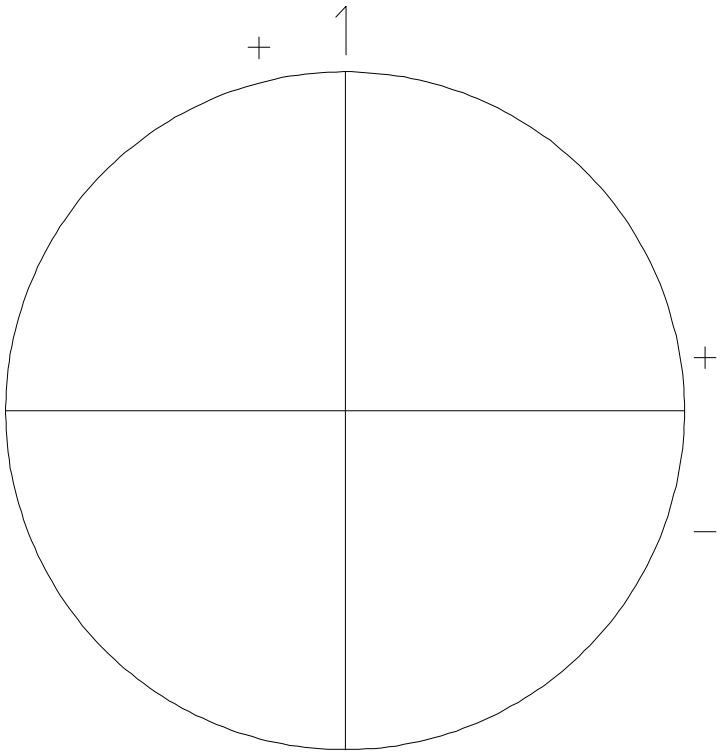
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

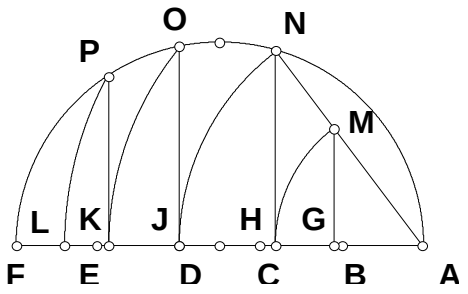
$\mathrm{mod}(7 + 1 - 1, 2) = 1$



Work in progress.

04/01/95 Exponential Series-Roots and Powers

I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.



$N_1 := 5$ $\quad AB := 1$ $\quad AF := AB \cdot N_1$

$\delta := 0 .. 3$

$N_2 := 3$ $\quad AJ_0 := AB \cdot N_2$ $\quad AN := AJ_0$

$$AJ_1 := \frac{AN^2}{AF} \qquad AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF}$$

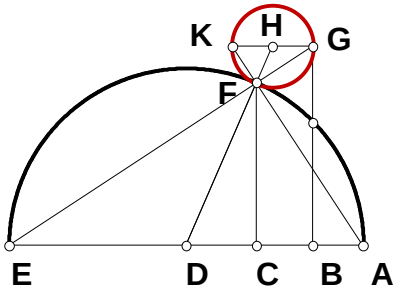
$AD_0 := AJ_0$ $\quad DF_0 := AF - AD_0$ $\quad DO_0 := \sqrt{AD_0 \cdot DF_0}$ $\quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{bmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{bmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

$$\sum_\delta \left[\frac{AF}{AJ_\delta} - \left(\frac{N_1}{N_2} \right)^{\delta+1} \right] = 0$$

$$\sum_\delta \left[\frac{AF}{AD_\delta} - \left(\frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$

09/13/95 A Study In Placement



Given AE, AB, AC what is GH?

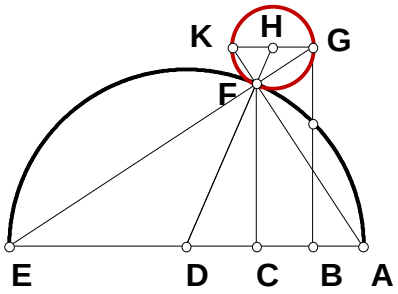
$N_1 := 1 \quad N_2 := 4 \quad N_3 := 1 \quad N_4 := 5 \quad AE := 1$

$AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad AC := AE \cdot \frac{N_3}{N_4} \quad DF := AD$

$DE := AD \quad CD := AD - AC \quad CF := \sqrt{DF^2 - CD^2}$

$CE := AE - AC \quad BD := AD - AB \quad BE := AE - AB \quad EF := \sqrt{CE^2 + CF^2} \quad EG := \frac{EF \cdot BE}{CE}$

$FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right| \quad GH - \left| \frac{N_3 \cdot N_2 - N_1 \cdot N_4}{2 \cdot N_2 \cdot (N_4 - N_3)} \right| = 0$



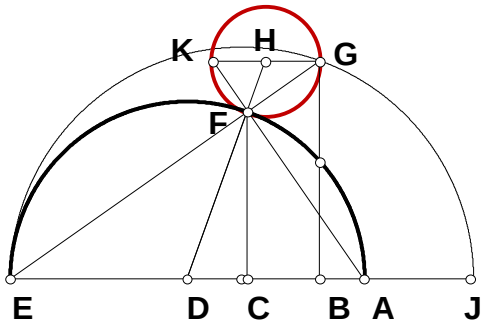
Given AE, AB, EF what is GH?

$AE := 1 \quad AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad DE := AD$

$EF := AE \cdot \frac{N_3}{N_4} \quad CE := \frac{EF^2}{AE} \quad BD := AD - AB$

$BE := BD + DE \quad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF$

$GH := \frac{AD \cdot FG}{EF} \quad GH - \frac{N_2 \cdot N_4^2 - N_1 \cdot N_4^2 - N_3^2 \cdot N_2}{2 \cdot N_2 \cdot N_3^2} = 0$



Given AE, AB, BG what is GH?

$AE := 1 \quad AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad BG := AE \cdot \frac{N_3}{N_4}$

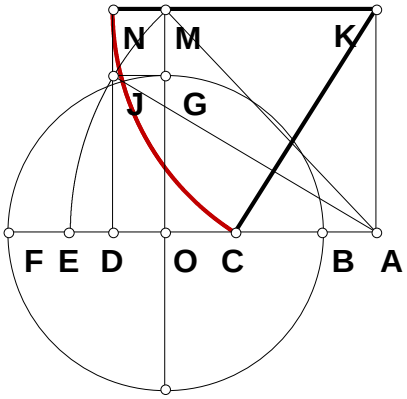
$BE := AE - AB \quad EG := \sqrt{BE^2 + BG^2} \quad EJ := \frac{EG^2}{BE}$

$EF := \frac{EG \cdot AE}{EJ} \quad FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right|$

$GH - \left| \frac{N_1 \cdot N_4^2 \cdot N_2 - N_1^2 \cdot N_4^2 - N_3^2 \cdot N_2^2}{2 \cdot N_2 \cdot \left[N_4^2 \cdot (N_1 - N_2) \right]} \right| = 0$

10/14/95 Alternate Method Square Root

For any AK is AC the root of AB x AF?



$N_1 := 5 \quad N_2 := 4$

$AB := 1 \quad AF := AB \cdot N_1 \quad AK := AB \cdot N_2 \quad BF := AF - AB$

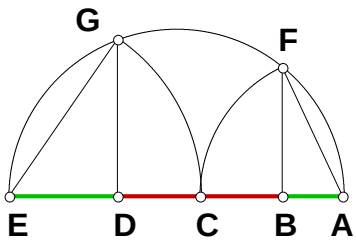
$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$GO := BO \quad DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$KN := AD \quad CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$

10/20/95 Four Times The Square



$N_1 := 2$ $AE := 1$ $AB := \frac{AE}{N_1}$

$BE := AE - AB$ $BF := \sqrt{AB \cdot BE}$ $AF := \sqrt{AB^2 + BF^2}$

$AC := AF$ $CE := AE - AC$ $EG := CE$ $DE := \frac{EG^2}{AE}$

$BD := AE - (AB + DE)$ $\frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$

$AD := AE - DE$ $DG := \sqrt{AD \cdot DE}$

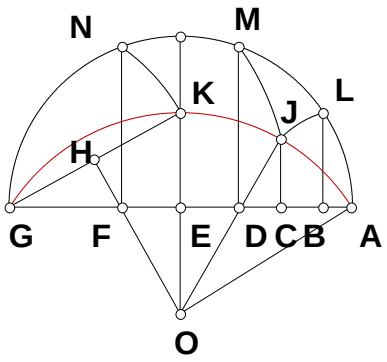
Algebraic Names:

$\frac{1}{N_1} - AB = 0$ $1 - \frac{1}{N_1} - BE = 0$ $\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0$ $\sqrt{\frac{N_1}{N_1^2}} - AF = 0$

$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0$ $1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0$ $2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$

$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0$ $\frac{\sqrt{2 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1^2 - 5 \cdot N_1 + 4 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1 - 1}}{N_1} - DG = 0$

11/01/95 A Modification Of The Square Root Figure, Gemini Roots



On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.

$N_1 := 9 \qquad N_2 := 2 \qquad AG := 1$

$AE := \frac{AG}{2} \quad EG := AE \quad EF := \frac{AG}{2 \cdot N_1} \quad AF := AE + EF$

$FG := EG - EIFN := \sqrt{AF \cdot FG} \quad GN := \sqrt{FN^2 + FG^2} \quad GK := GN$

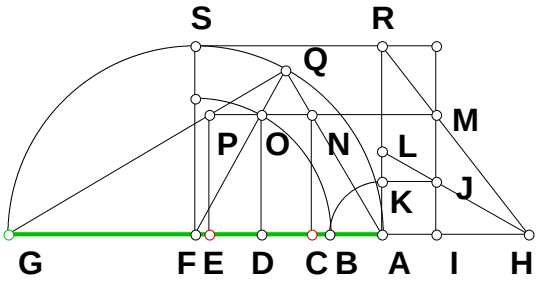
$EK := \sqrt{GK^2 - EG^2} \quad EO := \frac{EG \cdot EF}{EK} \quad OK := EO + EK \quad DE := \frac{AE}{N_2} \qquad DO := \sqrt{DE^2 + EO^2}$

$DJ := OK - DO \quad CD := \frac{DE \cdot DJ}{DO} \quad CE := CD + DE \; CJ := \frac{EO \cdot DJ}{DO} \quad AC := AE - CE \quad AJ := \sqrt{AC^2 + CJ^2}$

$AL := AJ \quad AB := \frac{AL^2}{AG} \quad CG := AG - AC \quad GJ := \sqrt{CG^2 + CJ^2} \quad GM := GJ \quad DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG) \qquad \frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$

11/05/95 Alternate Method Gemini Roots



$N_1 := 3 \quad N_2 := 5$

$AG := 1 \quad AF := \frac{AG}{2} \quad AR := AF \quad FQ := AF \quad FG := AF$

$AL := \frac{AR}{N_1} \qquad IM := \frac{AR}{N_2} \qquad AK := \frac{AL \cdot IM}{AR}$

$DO := IM \quad AB := AK \quad BF := AF - AB \quad FO := BF$

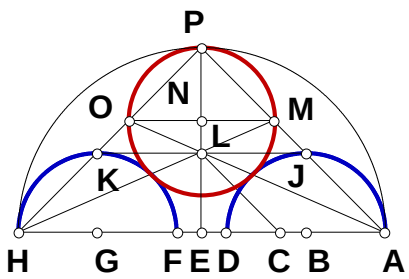
$OQ := FQ - FO \quad NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO} \quad NP - 2 \cdot AK = 0 \quad CD := AK \quad DE := AK$

$DF := \sqrt{FO^2 - DO^2} \quad AD := AF - DF \quad AC := AD - CD \quad EG := FG + DF - DE$

$CE := NP \qquad \frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$

12/01/95 Method For Equals

Given AB find NP.



$N_1 := 3$

$AH := 1 \quad AE := \frac{AH}{2} \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{AE}{N_1} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$

$$\frac{1}{2} \cdot \frac{\left(N_1 - 1\right)}{\left(N_1 + 1\right)} - NP = 0$$

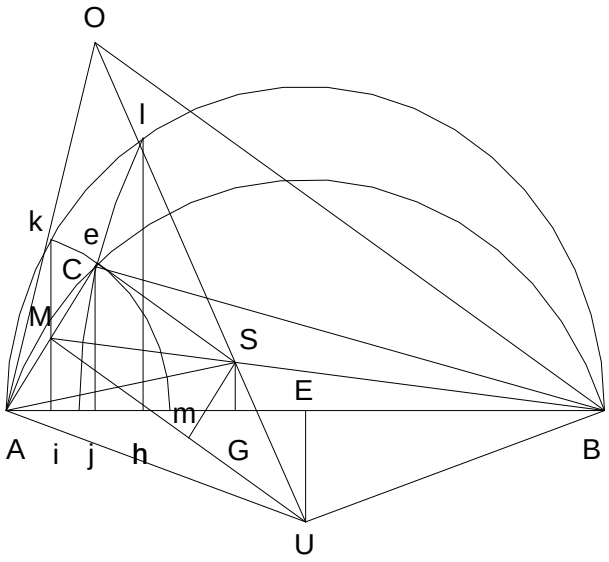
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2$ $AC := \begin{pmatrix} Side_1 \\ Side_2 \\ Side_3 \end{pmatrix}$ $BC := \begin{pmatrix} Side_2 \\ Side_3 \\ Side_1 \end{pmatrix}$ $AB := \begin{pmatrix} Side_3 \\ Side_1 \\ Side_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side_1 + Side_2 > Side_3)\cdot(Side_1 + Side_3 > Side_2)\cdot(Side_2 + Side_3 > Side_1)$



$AE_\delta := \frac{AB_\delta}{2}$ $Ak_\delta := AC_\delta$ $Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta}$ $Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$ $Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta$ $Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2}$ $BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta$ $Bg_\delta := \frac{BC_\delta}{2}$ $Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta$ $Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

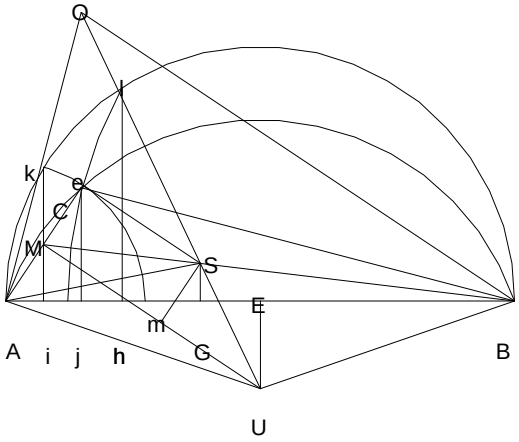
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2}$ $AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta}$ $BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2}$ $BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3}$ $BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta}$ $GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta$ $AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$



$MS_\delta := BM_\delta - BS_\delta$ $AU_\delta := BU_\delta$ $MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2}$ $Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

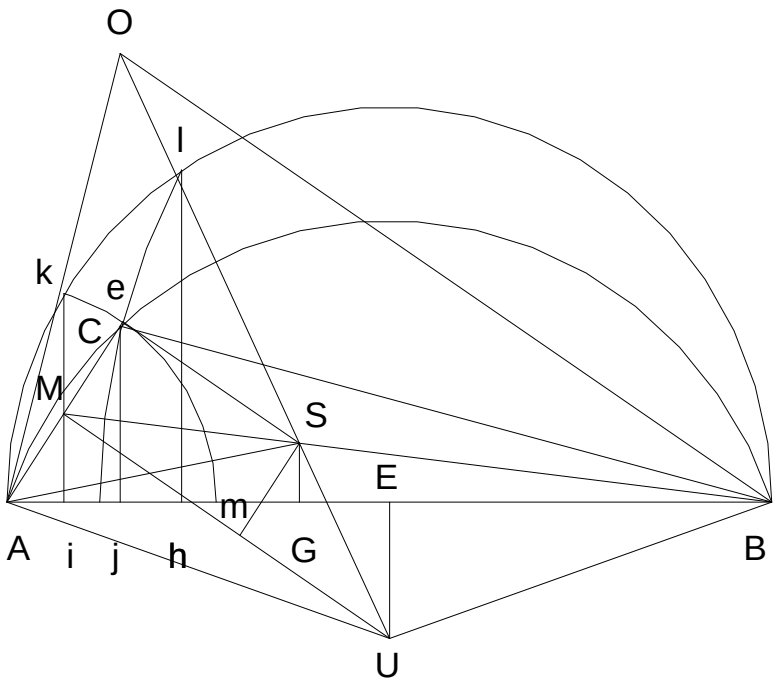
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

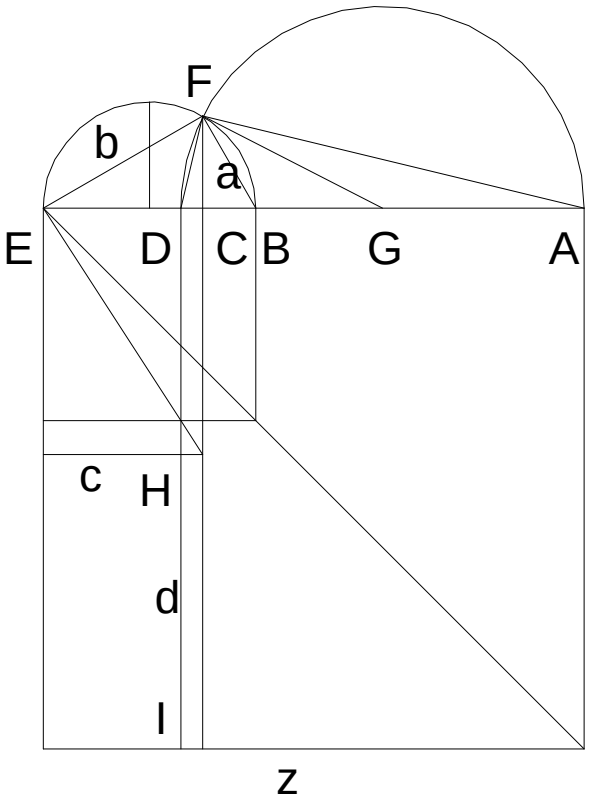


$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

0
1
1

SU_δ	UO_δ	AU_δ
14.15083	42.45249	16.70208
14.15083	42.45249	16.70208
14.15083	42.45249	16.70208

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.



12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a \qquad BE := \sqrt{a^2 + b^2} \qquad BC := \frac{a^2}{BE}$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2} \qquad CG := \sqrt{FG^2 - CF^2}$

$AG := FG \qquad AC := AG + CG$

$BG := CG - BC \qquad DG := FG$

$BD := DG - BG \qquad AB := AG + BG$

$AE := AB + BE \qquad DE := BE - BD$

$DH := \frac{b^2}{DE} \qquad DI := AE \qquad HI := DI - DH$

$z := AE \qquad z = 12.622$

$c := DE \qquad c = 0.622$

$d := HI \qquad d = 6.186$

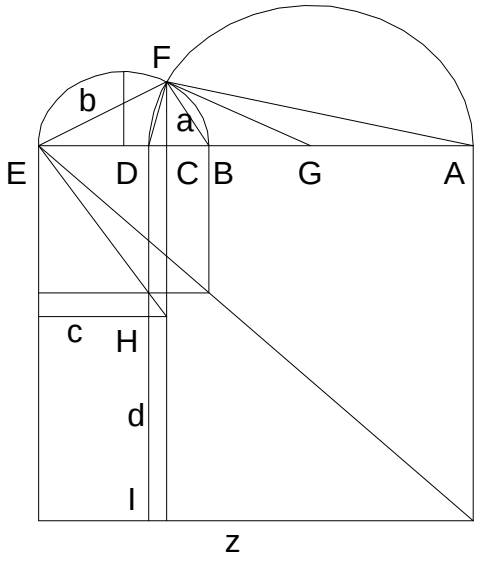
$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

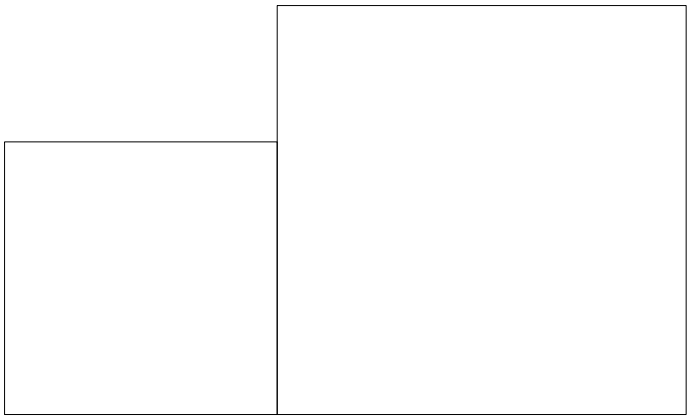
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$

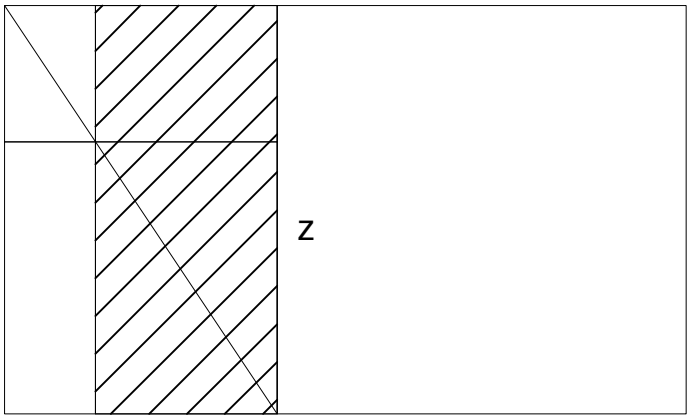


12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

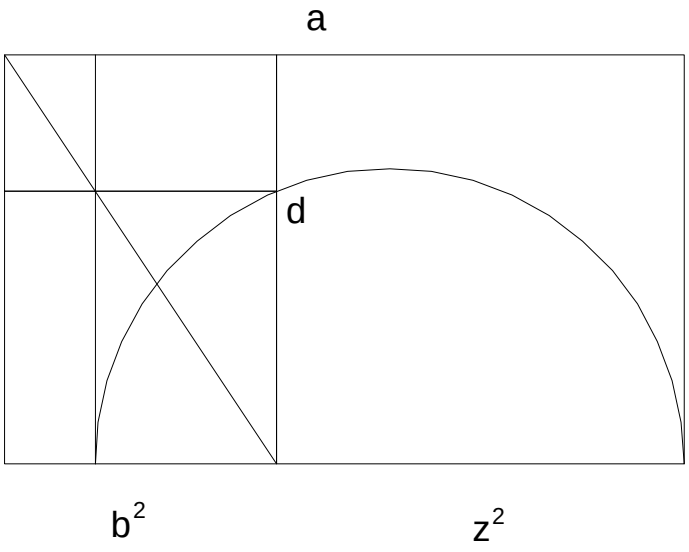


$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.



$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

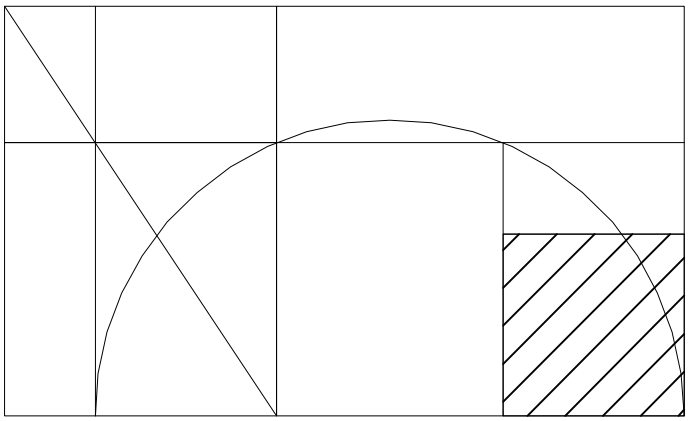
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

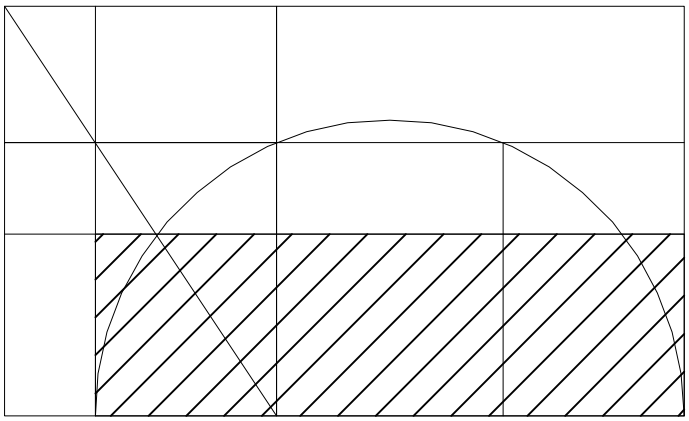
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

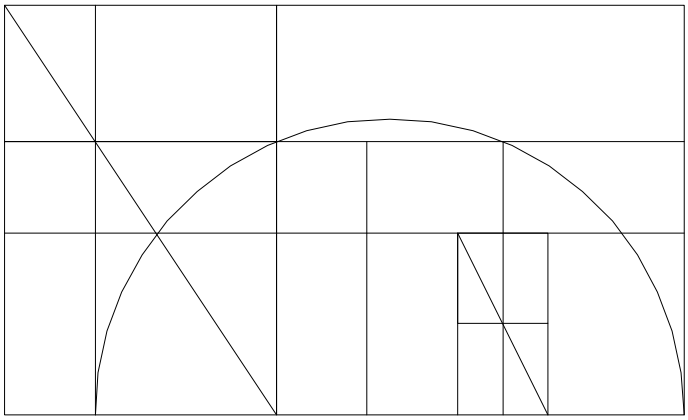
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



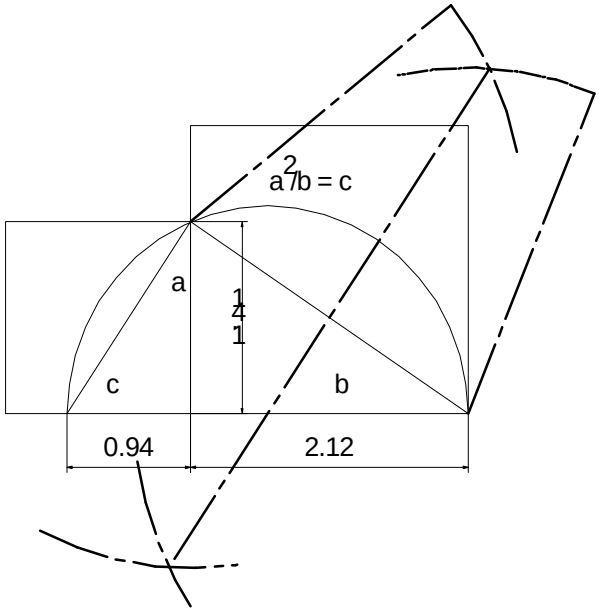
b^2 z^2



b^2 z^2

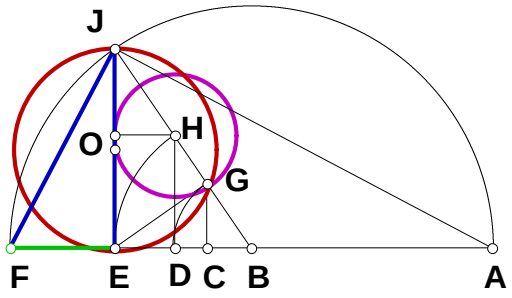


b^2 z^2



The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

12/20/95 Just For Fun



$N := 2$

$EF := 1 \quad EJ := EF \cdot N \quad AE := \frac{EJ^2}{EF} \quad AF := AE + EF$

$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF \quad BH := BE$

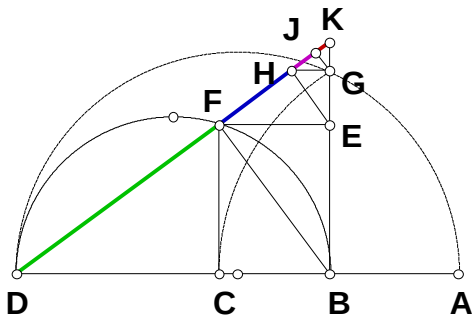
$BJ := \sqrt{EJ^2 + BE^2} \quad BD := \frac{BE \cdot BH}{BJ} \quad BG := BD$

$BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD \quad HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \quad \frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1 \quad \left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

$$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2+1\right)^2} = 0$$

12/21/95 Pascal's Triangle With Exponential Division



$N := 7$

$AB := 1 \qquad AD := AB \cdot N \qquad AC := \sqrt{AB \cdot AD}$

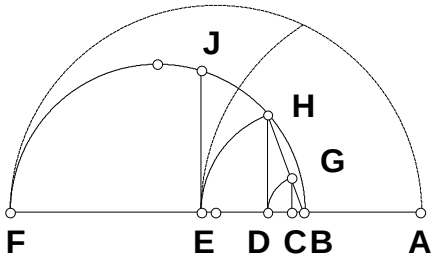
$BD := AD - AB \qquad CD := AD - AC \qquad BC := BD - CD$

$CF := \sqrt{BC \cdot CD} \qquad CF := \sqrt{BC \cdot CD} \qquad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \qquad FK := \frac{DK \cdot BC}{BD} \qquad HK := \frac{FK \cdot FK}{DK} \qquad JK := \frac{HK \cdot HK}{FK}$

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{DK}{JK} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

A more civil figure.

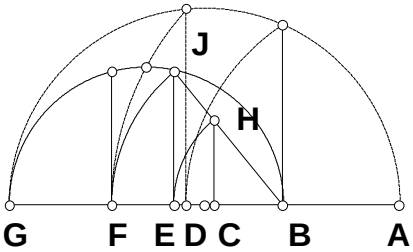


$AB := 1 \qquad AF := N \qquad BF := AF - AB \qquad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \qquad BH := BE \qquad BD := \frac{BE \cdot BH}{BF} \qquad BG := BD$

$BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$



$AB := 1 \qquad AG := N \qquad BG := AG - AB \qquad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}}$

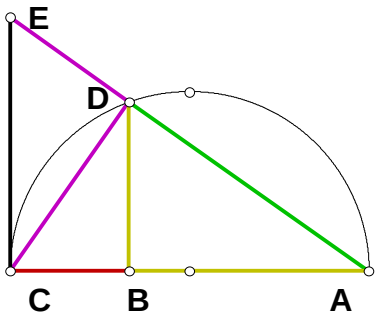
$BF := AF - AB \qquad BJ := BF \qquad BE := \frac{BJ \cdot BF}{BG} \qquad BH := BE$

$BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^2 - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0 \qquad \frac{BG}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$

12/29/95

Given AC and CE find BC when it is equal to $\frac{AC \cdot CE^2}{AC^2 + CE^2}$.



$N_1 := 1.854 \quad N_2 := 1.313$

$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$

$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$

$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \qquad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \qquad AB - \left(N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2}\right) = 0$

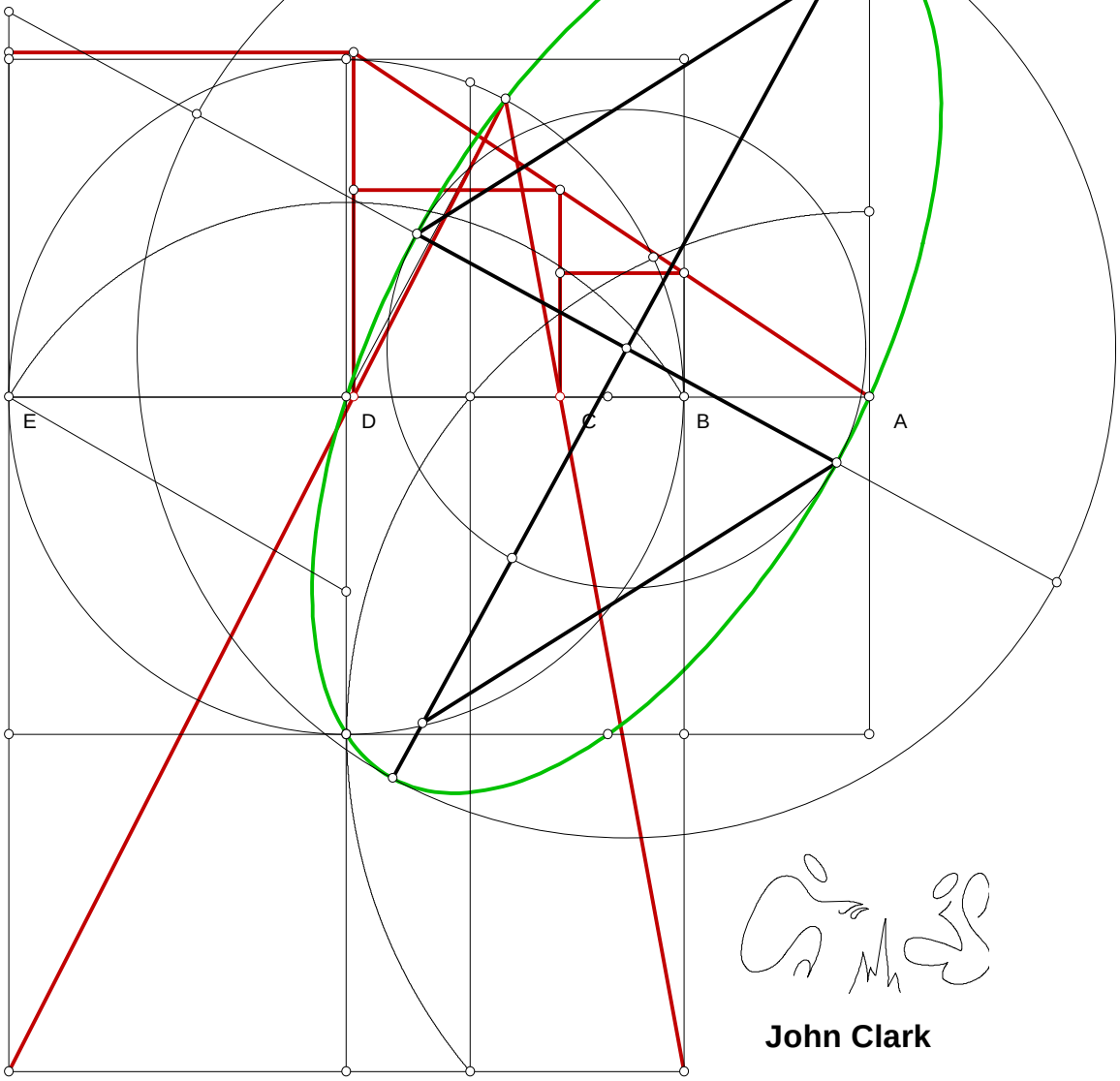
$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \qquad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

The Delian Quest

1996

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

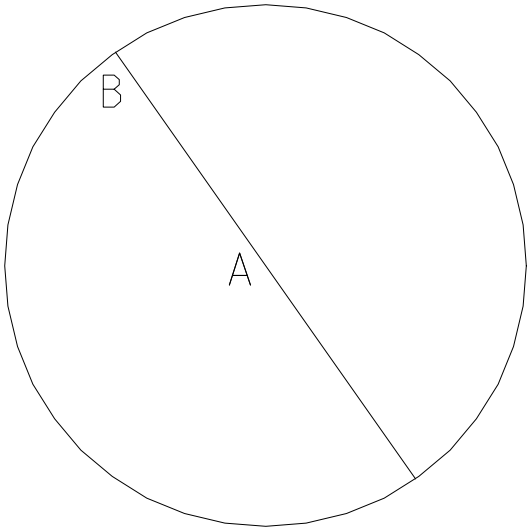


John Clark

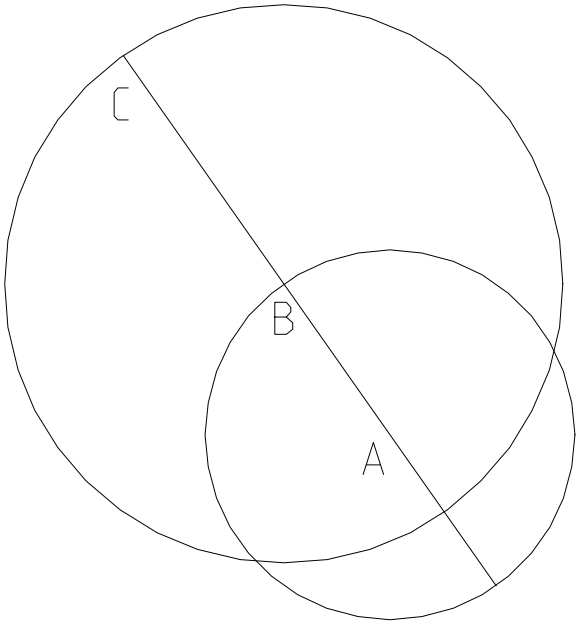
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

The Archamedian Paper Trisector- Without the Numbers.

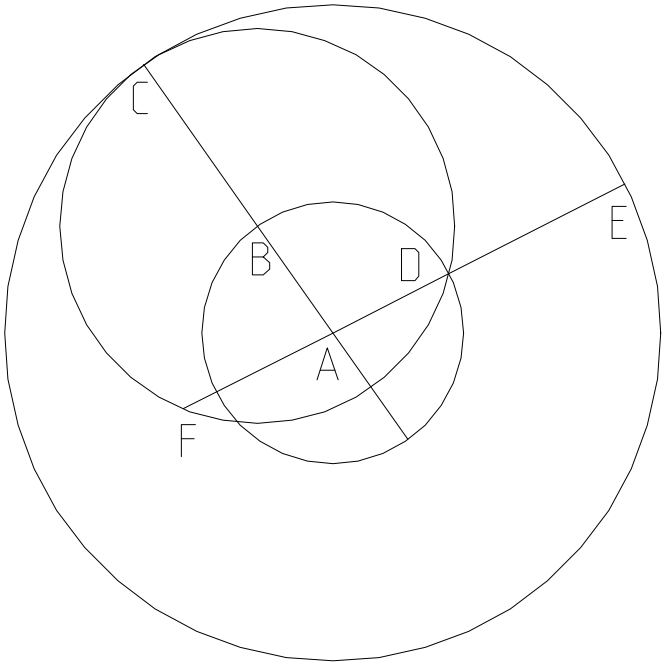
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.



Given any circle AB

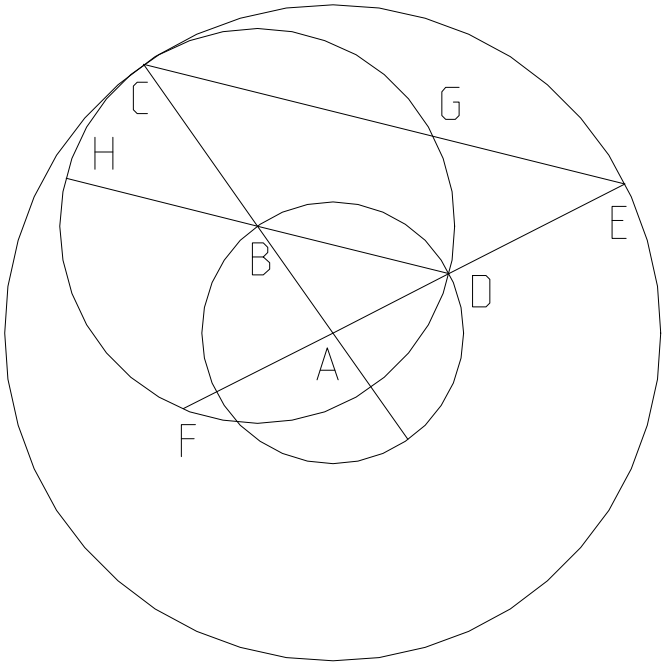


Given any circle BC such that BC≤ 2AB.

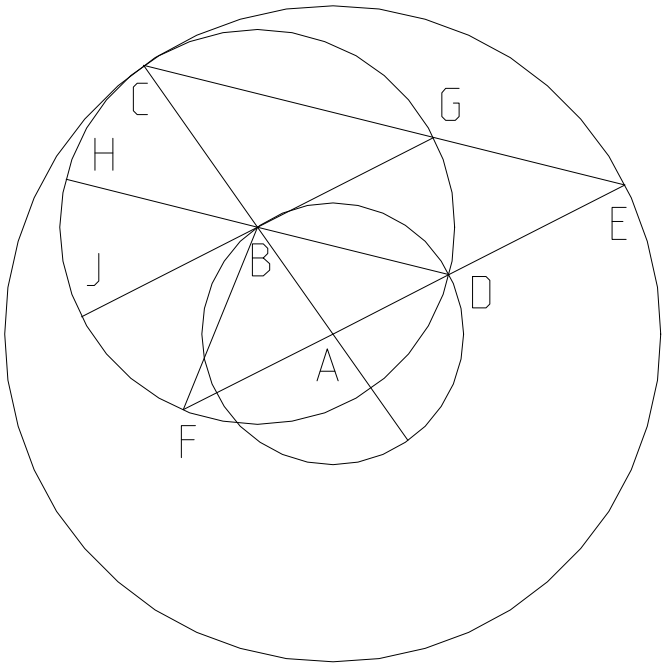


Construct AE such that AE = AC.

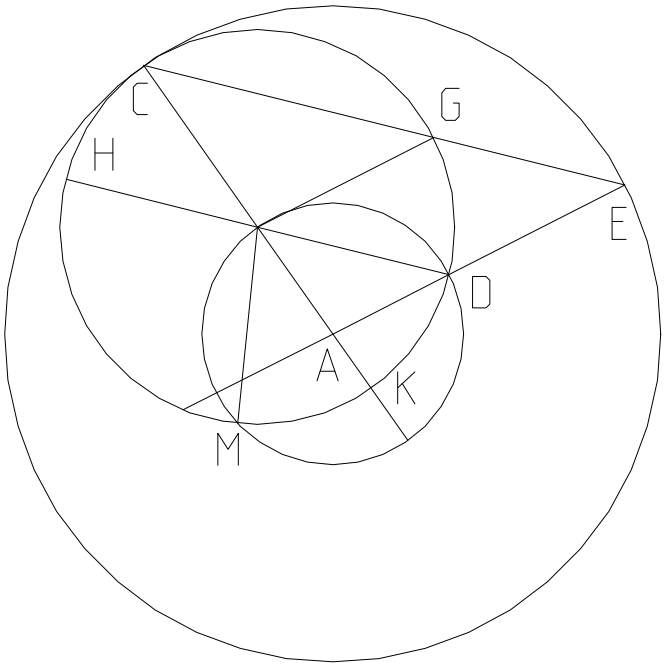
Since AC = AB+ BC and AD = AB, DE = BC.



Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



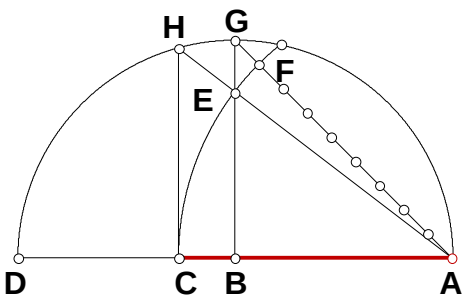
Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.



By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

01/07/96 Rusty Cubes



A rusty Compass construction for the duplication of the cube.

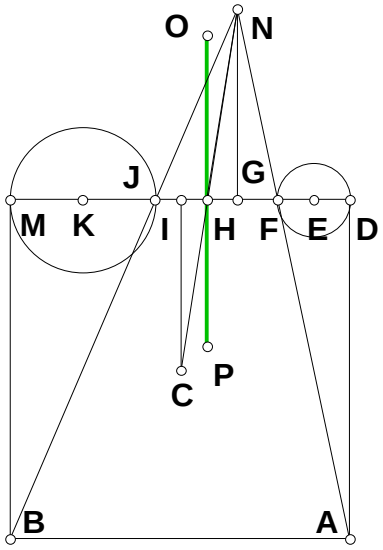
$AD := 2$ $AB := \frac{AD}{2}$ $AG := \sqrt{2 \cdot AB^2}$ $AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26$ $\frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.

01/08/96 Alternate Method Power Line



$R_1 := 2 \qquad R_2 := 5 \qquad D := 4$

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + JK) \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \qquad GJ := FJ - FG \qquad DI := \frac{DM}{2}$

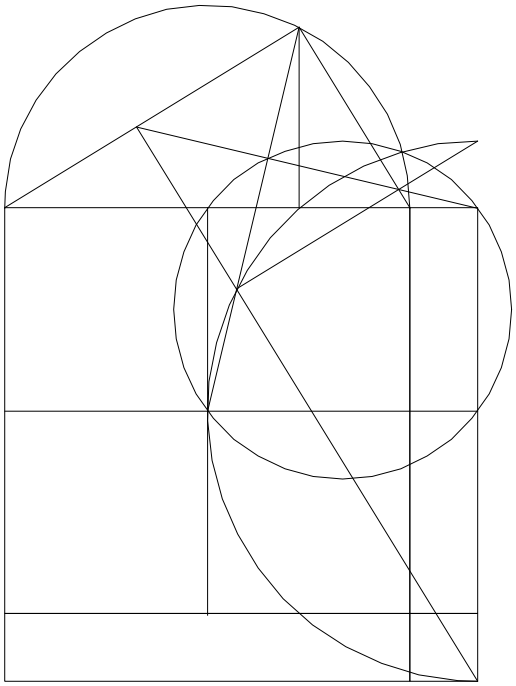
$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

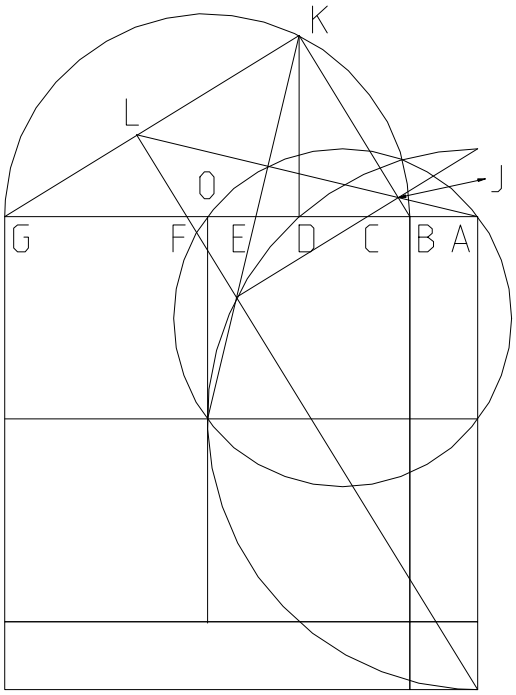
$HM := DM - DH$

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.



$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

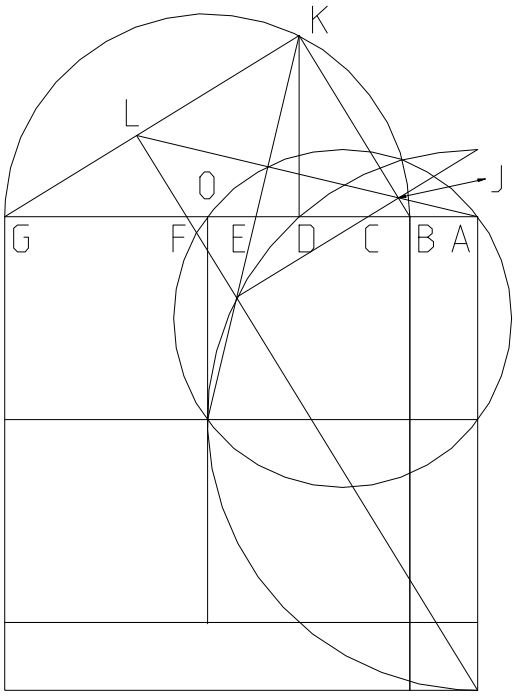
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



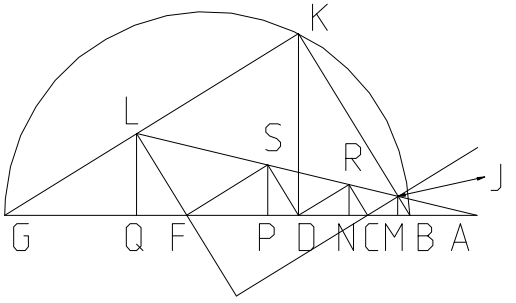
$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$



$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$$

$$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

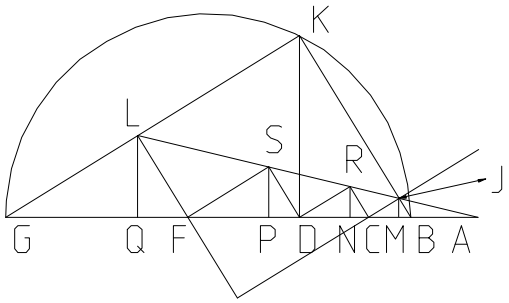
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$



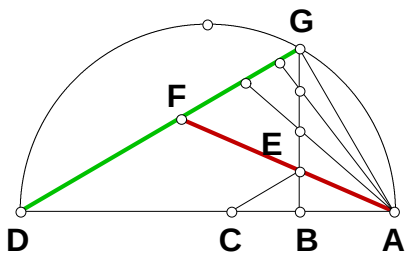
$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$

01/13/96 Pyramid of Ratios VI, Moving the Point



$N_1 := 5 \quad N_2 := 7 \qquad N_1 := N_1 \quad N_2 := N_2$

$AD := 1 \quad AB := \frac{AD}{N_1} \qquad BD := AD - AB$

$BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG}{N_2} \qquad BC := \frac{BD \cdot BE}{BG}$

$AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \quad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1)\cdot(N_2 - 1)} = 0$

$\Delta := 2 .. N_1 \qquad \delta := 2 .. N_2$

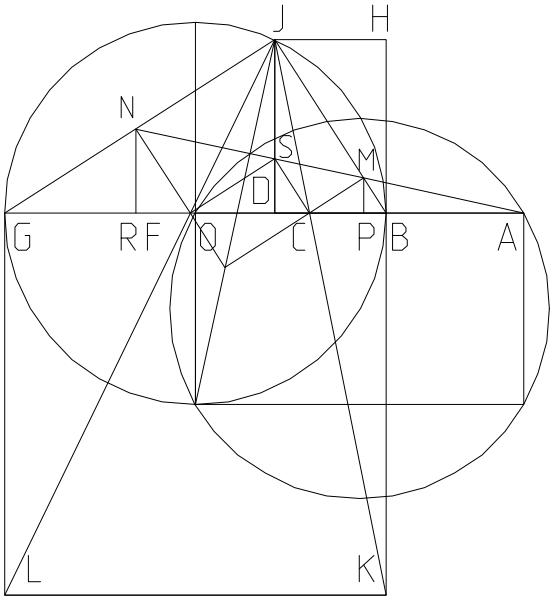
$SeriesAF_{\Delta,\delta} := \frac{\Delta \cdot \delta}{(\Delta - 1)\cdot(\delta - 1)} \qquad SeriesAF = \begin{bmatrix} 4 & 3 & 2.667 & 2.5 & 2.4 & 2.333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.667 & 2 & 1.778 & 1.667 & 1.6 & 1.556 \\ 2.5 & 1.875 & 1.667 & 1.563 & 1.5 & 1.458 \end{bmatrix}$

$DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2} \quad DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF$

$\frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$

$SeriesDG_{\Delta,\delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)} \qquad SeriesDG = \begin{bmatrix} 3 & 2 & 1.667 & 1.5 & 1.4 & 1.333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333 & 2 & 1.8 & 1.667 \\ 6 & 3.5 & 2.667 & 2.25 & 2 & 1.833 \end{bmatrix}$

The figure cuts the base in Cube Roots and provides some interesting ratios.



$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

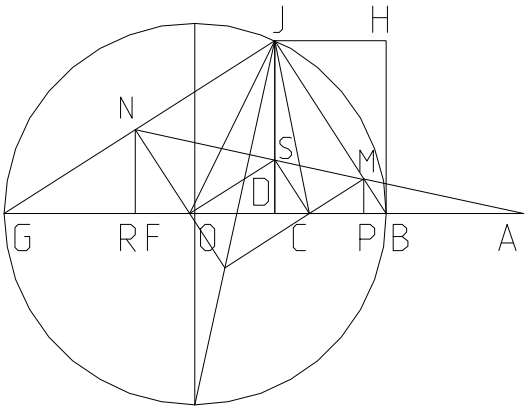
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$



$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

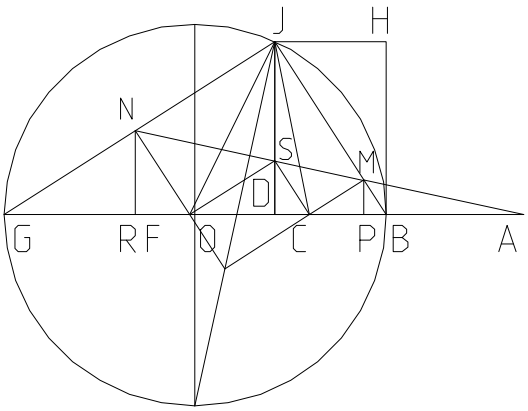
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$



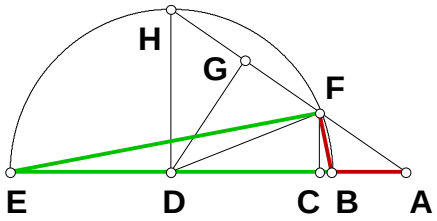
$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$

01/17/96 A Right Triangle In A Given Ratio

Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.



$N := 3$

$AB := 1 \quad AE := AB \cdot N \quad BE := AE - AB$

$BD := \frac{BE}{2} \quad DF := BD \quad DE := BD \quad AD := AB + BD$

$DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$S_1 := AD \quad S_2 := AF \quad S_3 := DF$

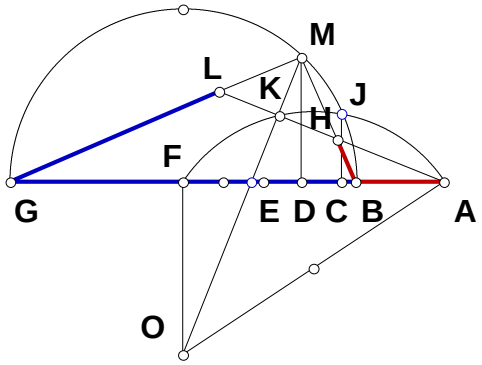
$CD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad BC := BD - CD$

$CE := CD + DE \quad CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2} \quad AC := AD - CD$

$\frac{AE}{AB} - \frac{EF}{BF} = 0 \qquad AC - \frac{N^2 + N}{N^2 + 1} = 0 \qquad BF - \frac{N - 1}{\sqrt{N^2 + 1}} = 0 \qquad EF - \frac{N^2 - N}{\sqrt{N^2 + 1}} = 0$

01/17/96B Divide The Sides Of A Triangle In A Given Ratio

Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.



$N_1 := 5 \qquad N_2 := 9$

$AB := 1 \qquad AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \frac{BG}{2}$

$AC := \frac{N_1^2 + N_1}{N_1^2 + 1} \qquad AF := AB + BF \quad CF := AF - AC$

$BC := AC - AB \quad BD := \frac{CF}{N_2} + BC \qquad DF := BF - BD$

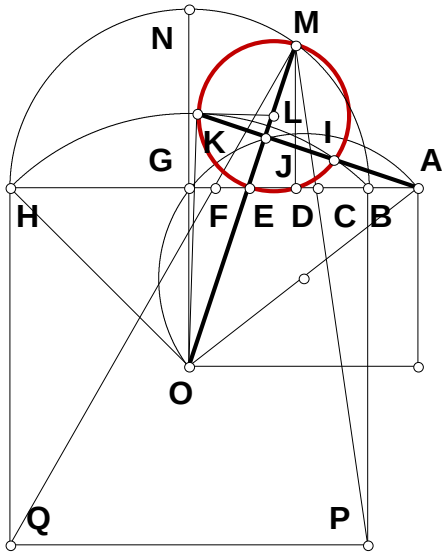
$DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF \quad EF := \frac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \frac{EO \cdot (DM + FO)}{FO} \quad EK := \frac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO) HK := KM \quad HM := \sqrt{2 \cdot KM^2}$

$BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM \; GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM$

$\frac{AG}{AB} - \frac{GL}{BH} = 0$

01/21/96 More On Cube Roots



$N := 5$ $BH := 1$

$BP := BH$ $HQ := BH$

$BG := \frac{BH}{2}$ $GO := BG$ $GN := BG$ $NO := BH$ $GH := BG$

$BE := \frac{BG}{N}$ $EG := BG - BE$ $EO := \sqrt{EG^2 + GO^2}$

$MO := \frac{GO \cdot NO}{EO}$ $EM := MO - EO$ $EL := \frac{EM}{2}$ $LK := EL$

$LO := EO + EL$ $LJ := \frac{LK^2}{LO}$ $EJ := EL - LJ$

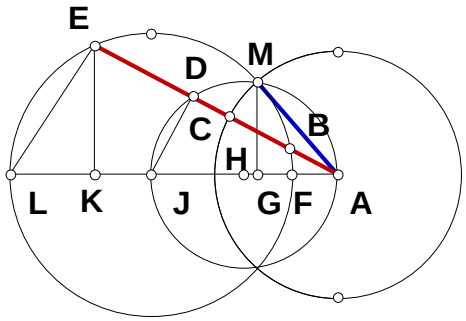
$AE := \frac{EO \cdot EJ}{EG}$ $AH := AE + EG + GH$ $AB := AH - BH$

$DE := \frac{EG \cdot EM}{EO}$ $DM := \frac{GO \cdot EM}{EO}$ $BD := BG - (EG + DE)$

$BC := \frac{BD \cdot BP}{BP + DM}$ $DH := BH - BD$ $DF := \frac{DH \cdot DM}{DM + HQ}$ $AC := AB + BC$ $AF := AB + BD + DF$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$

01/22/96 Trivial Method Square Root



For any E between M and L, AM is the square root of AB x AE.

$N_1 := 6 \qquad N_2 := 3$

$AF := 1 \quad AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

$GL := AL - AG \quad GK := \frac{GL}{N_2} \qquad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

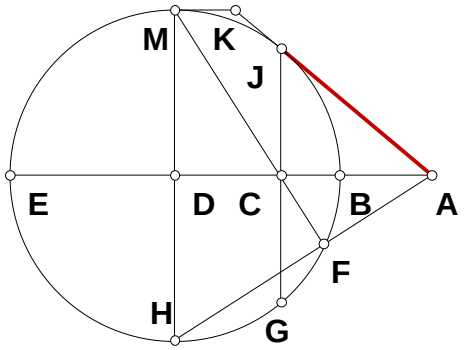
$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

01/24/96 Tangent

The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.



$N := 5 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2} \qquad AD := AB + BD$

$DM := BD \qquad DH := BD \qquad CD := \frac{DH \cdot DM}{AD}$

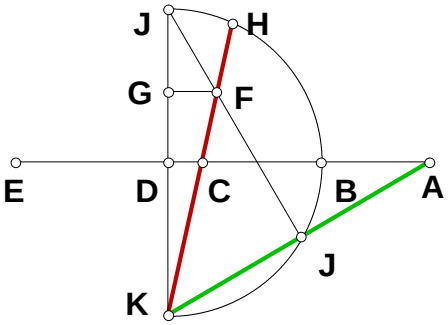
$BC := BD - CD \qquad CE := BE - BC$

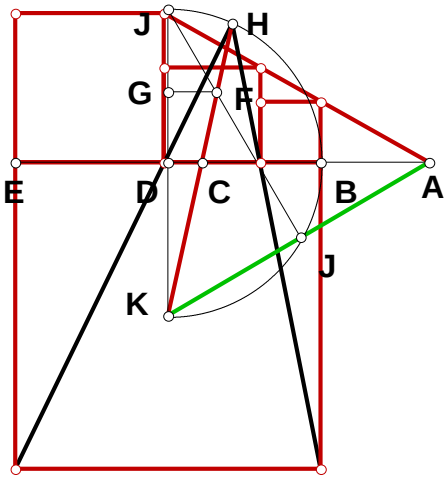
$CJ := \sqrt{BC \cdot CE} \qquad AJ := \sqrt{(AB + BC)^2 + CJ^2}$

$AJ - \sqrt{N} = 0$

$AC := AB + BC \qquad 2 \cdot AB \cdot \frac{N}{(1 + N)} - AC = 0$

01/25/96 On Cubes





Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.

$N := 9 \qquad BE := 1$

$BD := \frac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \frac{BD}{N} \qquad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$

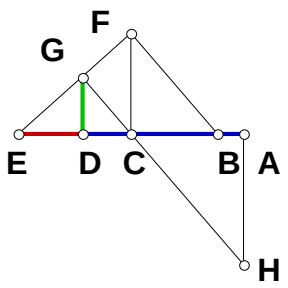
$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

$AB := AE - BE$

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

01/29/96 Linear division $\frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)}$



$N_1 := 3 \qquad N_2 := 9 \qquad AE := 1$

$AH := AE \cdot N_1 \quad AC := \frac{AE}{2} \quad CF := AE \cdot N_2 \quad BC := \frac{AC \cdot CF}{AH}$

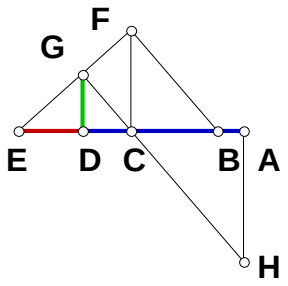
$CE := AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{AH \cdot CD}{AC}$

$DE - \frac{N_1}{2 \cdot (N_1 + N_2)} = 0 \qquad AD - \frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)} = 0 \qquad CD - \frac{N_2}{2 \cdot (N_1 + N_2)} = 0$

$DG - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

Linear division $\frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)}$



$N_1 := 4 \qquad N_2 := 4 \qquad N_3 := 2 \qquad AE := 1$

$AH := AE \cdot N_1 \quad AC := \frac{AE}{N_2} \quad CF := AE \cdot N_3 \quad BC := \frac{AC \cdot CF}{AH}$

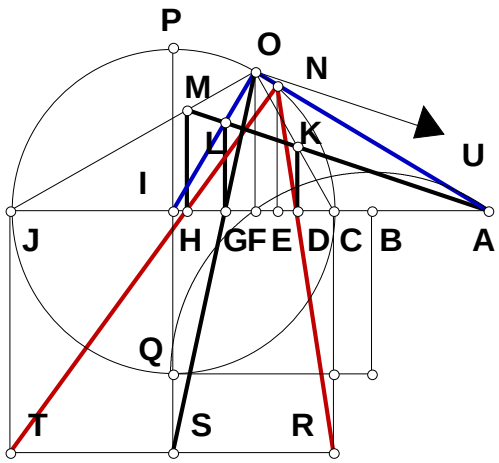
$CE := AE - AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{CF \cdot CE}{BE}$

$DE - AE \cdot \frac{N_1 \cdot (N_2 - 1)^2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0 \qquad AD - AE \cdot \frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

$DG - AE \cdot \frac{N_1 \cdot N_3 \cdot (N_2 - 1)}{N_1 \cdot N_2 - N_1 + N_3} = 0 \qquad CD - AE \cdot \frac{N_3 \cdot (N_2 - 1)}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

01/31/96 On Gemini Roots



Hitting AO from any RT while maintaining Gemini Roots.

$N_1 := 5 \qquad N_2 := 3 \qquad BC := 1$

$BJ := BC \cdot N_1 \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF$

$FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2 \quad HS := CR$

$FI := FJ - IJ \quad FG := \frac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \frac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \frac{HS \cdot GO}{OS} \quad FU := \frac{AG \cdot FO}{GL}$

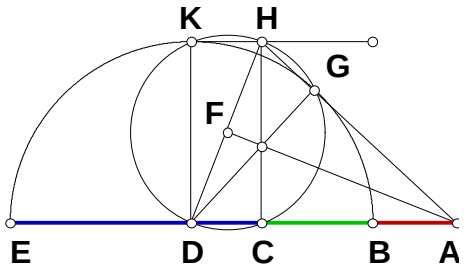
$AH := \frac{FU \cdot AJ}{FU + FJ} \quad DK := \frac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \frac{AG \cdot DK}{GL} \quad AC := AF - CF \quad CD := AD - AC$

$CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \frac{CR \cdot DH}{CD + HJ} \quad CE := \frac{CD \cdot (CR + EN)}{CR}$

$AE := AC + CE \quad \frac{AF}{FO} - \frac{AE}{EN} = 0 \quad BD := BC + CD \quad BH := BC + CH \quad \sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$

02/02/96 Find A Segment

Given BE and BC such that $\sqrt{(AB + BE)\cdot AB}$ = AB + BC, find AB.

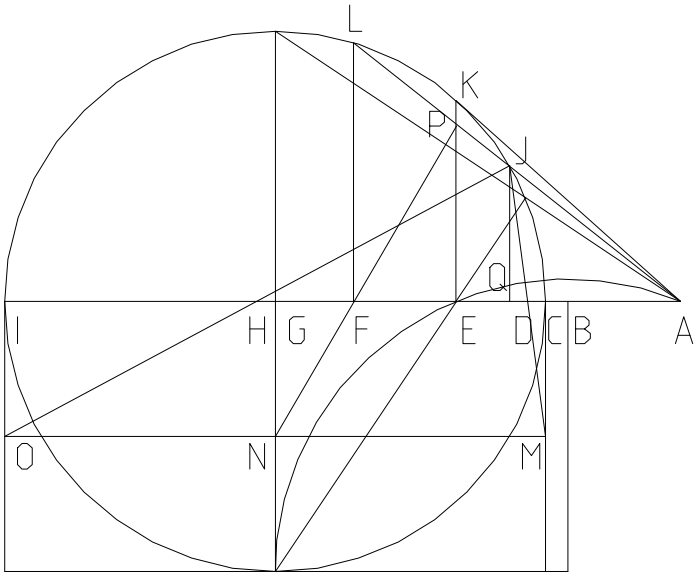


$N_1 := 5$ $N_1 > 2 = 1$

$BC := 1$ $BE := BC \cdot N_1$ $BD := \frac{BE}{2}$ $CD := BD - BC$

$CH := BD$ $DH := \sqrt{CD^2 + CH^2}$ $DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD}$ $AB := AD - BD$ $AB - \frac{1}{(N_1 - 2)} = 0$ $\sqrt{(AB + BE)\cdot AB} - (AB + BC) = 0$



Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

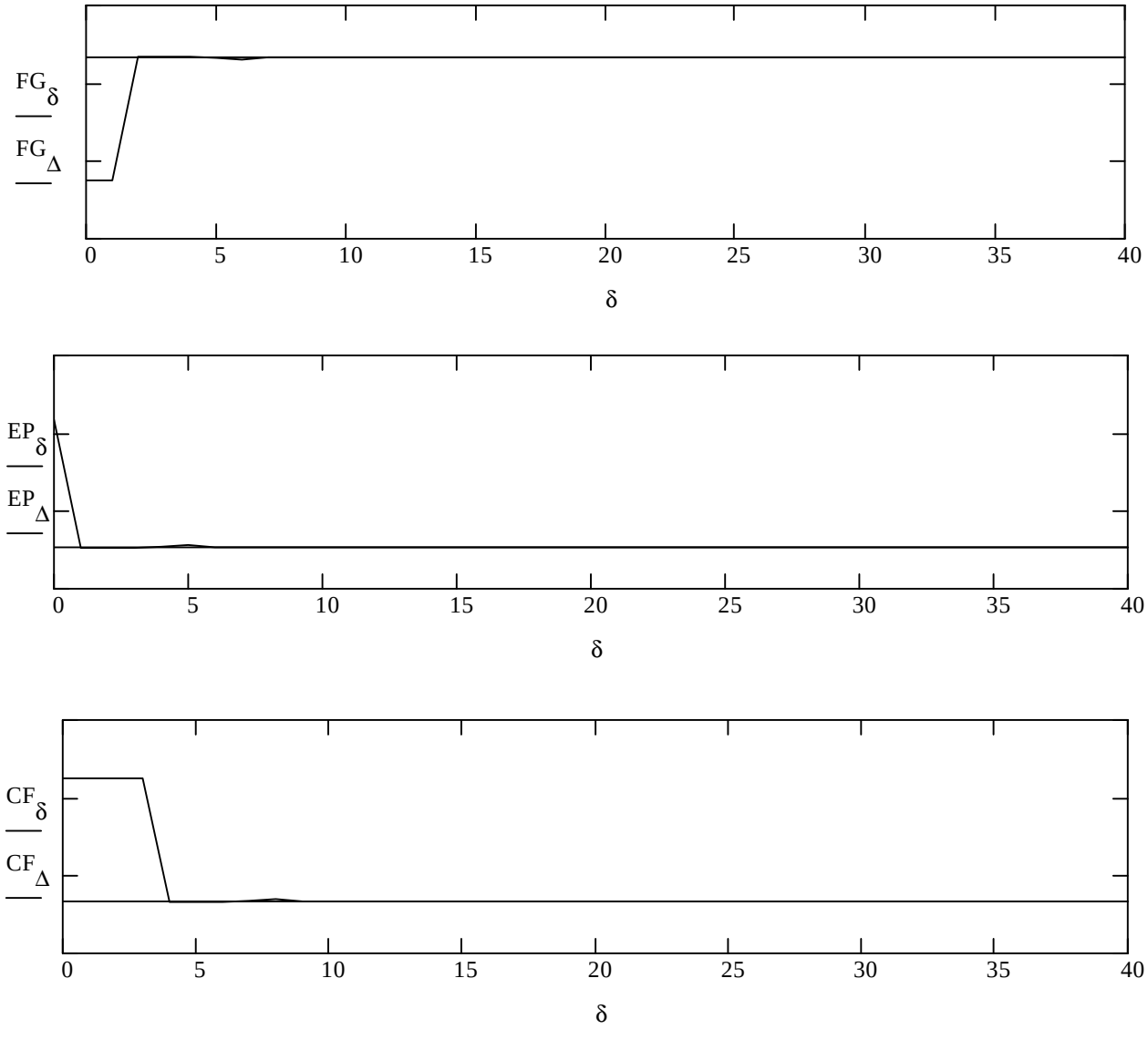
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000$ $\left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$

$BD = 1.259921049894873$ $2^{\frac{1}{3}} = 1.259921049894873$

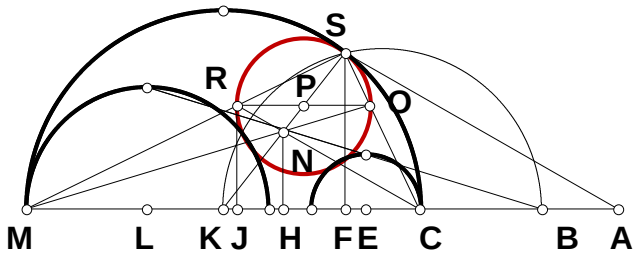
$BH = 1.587401051968199$ $4^{\frac{1}{3}} = 1.587401051968199$



The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.

04/14/96 Method for Unequals

Given three circles in the said configuration, find the fourth. I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."



$N_1 := 7$ $N_2 := 3$

$CM := 1$ $CK := \frac{CM}{2}$ $CE := \frac{CM}{N_1}$

$LM := \frac{CM}{N_2}$ $EL := CM - (CE + LM)$

$BL := \frac{EL \cdot LM}{LM - CE}$ $BM := BL + LM$ $BC := BM - CM$ $BK := \frac{CM}{2} + BC$

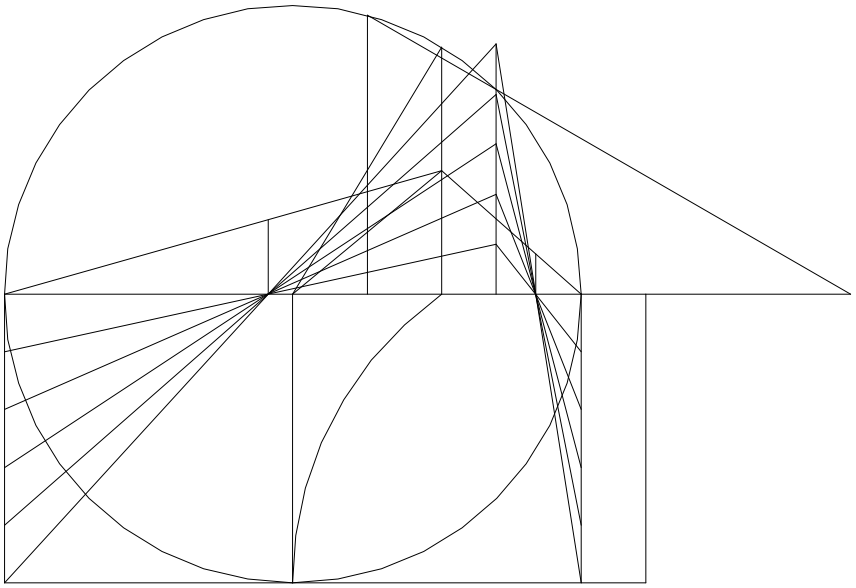
$R_1 := LM$ $R_2 := CE$ $D := EL$ $KS := CK$ $EH := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$

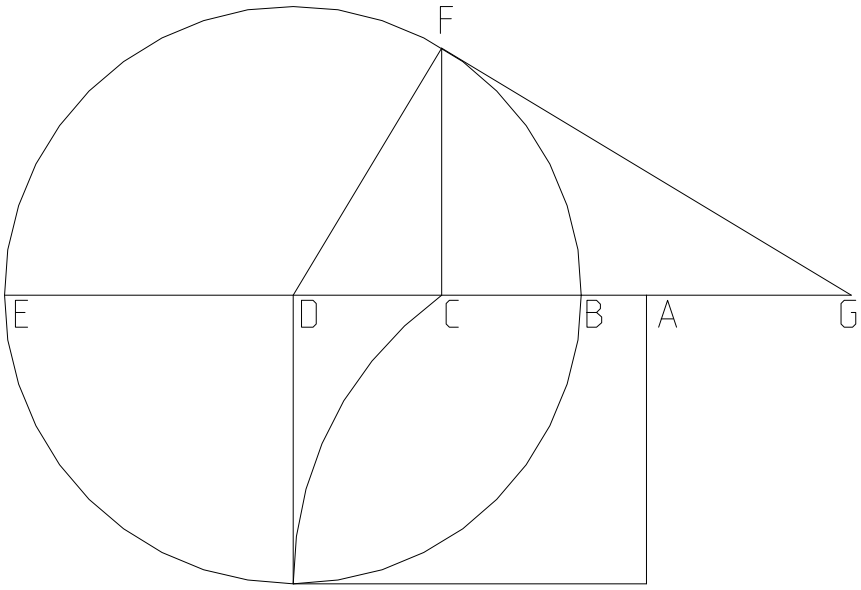
$FK := \frac{KS^2}{BK}$ $CF := CK - FK$ $FM := CM - CF$ $FS := \sqrt{CF \cdot FM}$

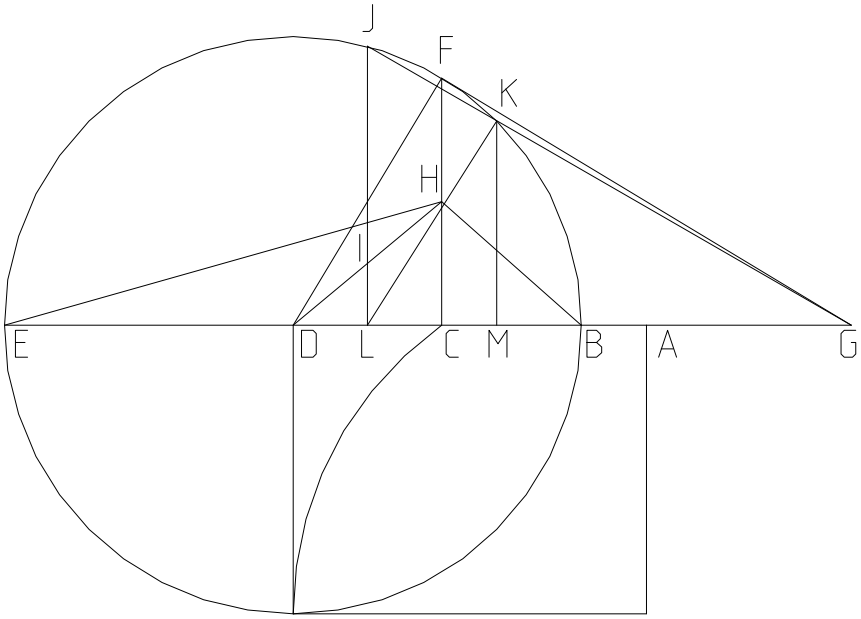
$HK := CK - (CE + EH)$ $CH := CK - HK$ $HN := \frac{FS \cdot HK}{FK}$ $AF := \frac{CH \cdot FS}{HN}$ $JR := \frac{FS \cdot CM}{AF + FM}$

$RO := \frac{CM \cdot (FS - JR)}{FS}$ $PS := \frac{RO}{2}$ $PS - \frac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$

On Gemini Roots







$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

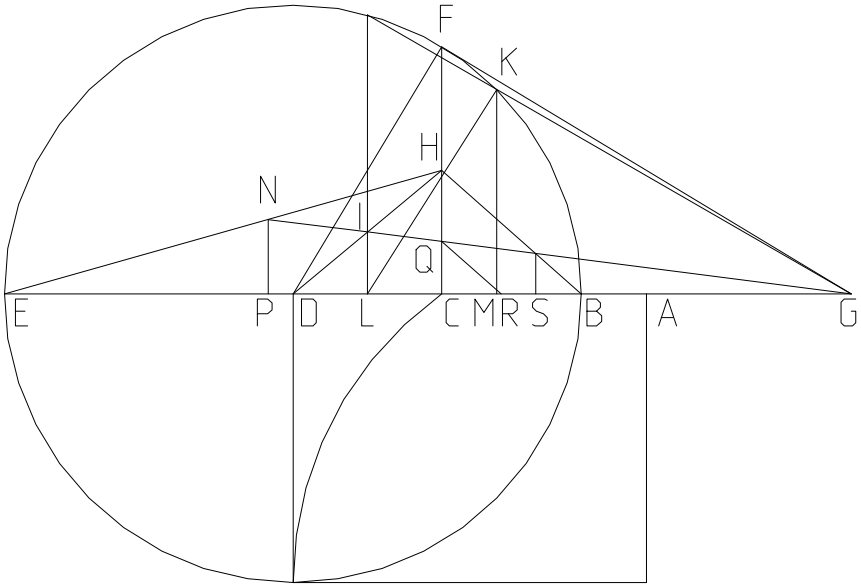
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

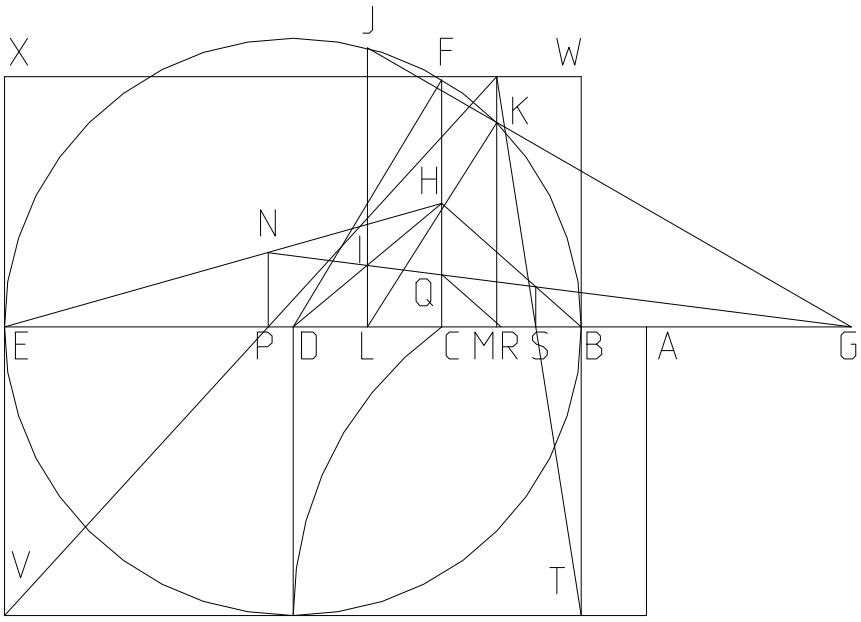


$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

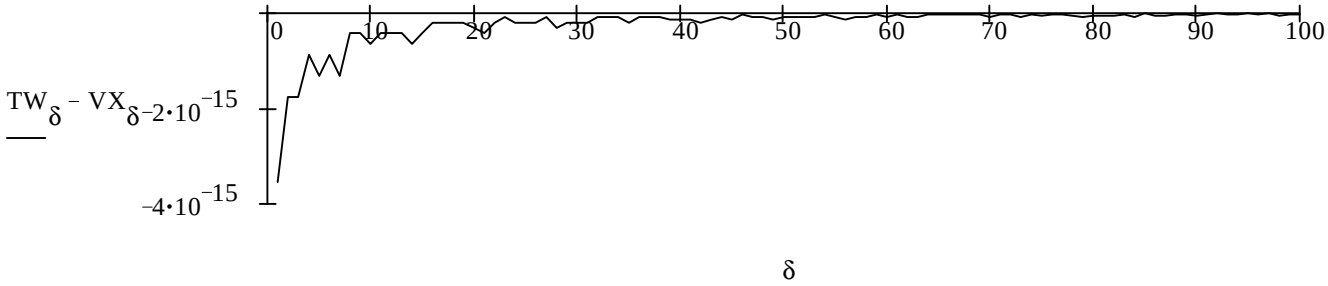
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$



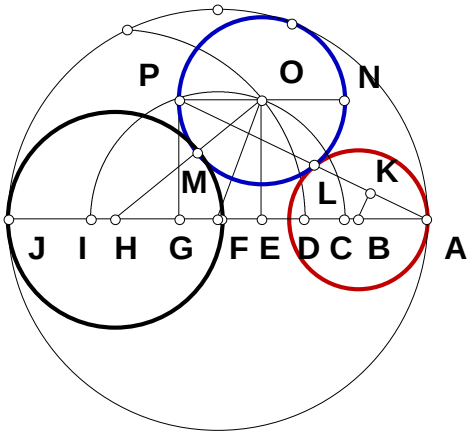
$$\delta := 1..100$$

$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$



04/16/96 Given Three Radii



Given AJ, HJ and NO find AB such that AB is collinear with AJ and HJ.

$N_1 := 4 \qquad N_2 := 16$

$AJ := 1 \qquad AF := \frac{AJ}{2} \qquad HJ := \frac{AJ}{N_1} \qquad NO := \frac{AJ}{N_2}$

$HM := HJ \quad MO := NO \quad HO := HM + MO$

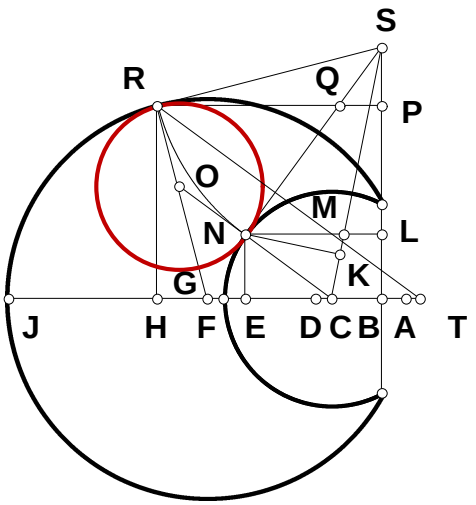
$FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$S_1 := FH \quad S_2 := FO \quad S_3 := HO \quad EH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO \quad AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP}$

$AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG} \qquad AB - \frac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot (N_2 \cdot N_1 - 2 \cdot N_2 - 4)} = 0$

04/17/96 A Circle In A Crescent



$N_1 := 2$ $N_2 := 2$ $N_3 := 3$

$AJ := 1$ $AG := \frac{AJ}{2}$ $FR := AG$ $AF := \frac{AJ}{N_1}$

$AD := \frac{AF}{2}$ $AC := \frac{AD}{N_2}$ $CF := AF - AC$

$CG := AG - AC$ $R_1 := AG$ $R_2 := CF$ $D := CG$

$BC := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$ $AB := AC + BC$

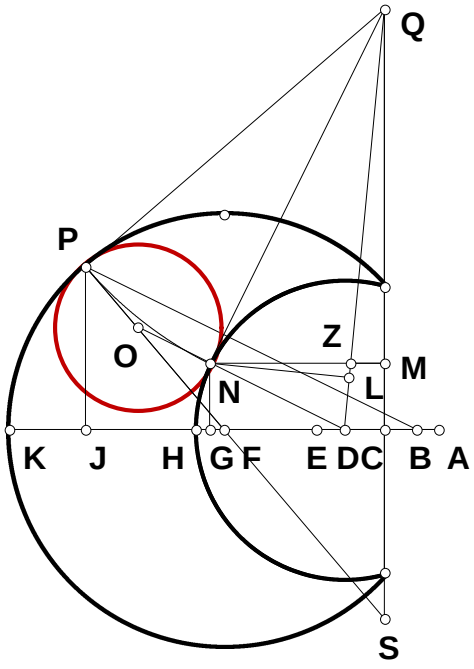
$BJ := AJ - AB$ $BH := \frac{BJ}{N_3}$ $AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{FR^2 - GH^2}$

$AP := HR$ $PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := AP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$ $KN := \sqrt{CN^2 - CK^2}$ $KM := \frac{BC \cdot KN}{BS}$

$SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$ $BL := BS - SL$ $EN := BL$ $CE := \sqrt{CN^2 - EN^2}$

$HT := \frac{CE \cdot HR}{EN}$ $GT := HT - GH$ $GO := \frac{FR \cdot CG}{GT}$ $OR := FR - GO$ $OR = 0.125$



04/17/96b A Circle In A Crescent

$N_1 := 1.7 \quad N_2 := 2 \quad N_3 := 1.3$

$AK := 1 \quad AF := \frac{AK}{2} \quad FP := AF \quad FK := AF$

$AH := \frac{AK}{N_1} \quad AE := \frac{AH}{2} \quad EH := AE \quad AD := \frac{AE}{N_2}$

$DE := AE - AD \quad DH := EH + DE \quad DN := DH$

$DF := AF - AD \quad R_1 := AF \quad R_2 := DH \quad D := DF$

$CD := \frac{R_2^2 + D^2 - R_1^2}{2 \cdot D} \quad AC := AD + CD$

$CK := AK - AC \quad CJ := \frac{CK}{N_3} \quad CF := CK - FK$

$FJ := CJ - CF \quad JP := \sqrt{FP^2 - FJ^2} \quad FS := \frac{FP \cdot CF}{FJ} \quad PS := FS + FP \quad QS := \frac{FP \cdot PS}{JP} \quad PQ := \frac{FJ \cdot QS}{FP}$

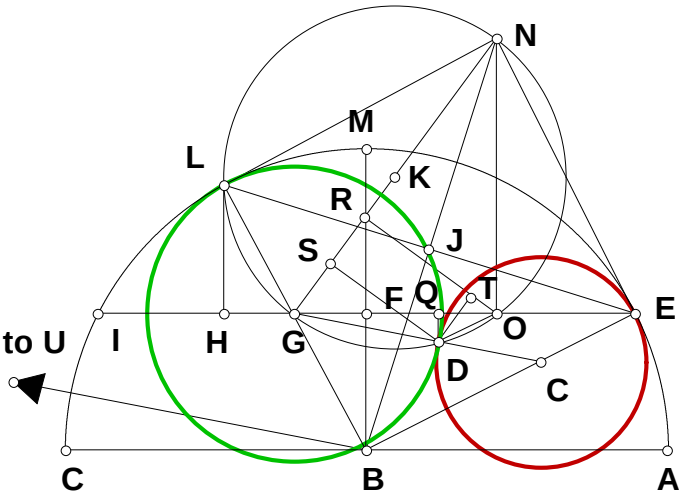
$CS := \frac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad NQ := PQ \quad DQ := \sqrt{NQ^2 + DN^2} \quad DL := \frac{DN^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2}$

$LZ := \frac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ \quad MQ := \frac{CQ \cdot QZ}{DQ} \quad CM := CQ - MQ \quad GN := CM$

$CG := \sqrt{DN^2 - GN^2} \quad BJ := \frac{CG \cdot JP}{GN} \quad BF := BJ - FJ \quad FO := \frac{FP \cdot DF}{BF} \quad OP := FP - FO \quad OP = 0.175$

04/22/96

Given BF as a ratio to BM and EG as a ratio to EI, what is CE?



$N_1 := 2.5$ $N_2 := 1.5$ $AC := 1$

$AB := \frac{AC}{2}$ $BM := AB$ $BE := AB$

$BL := AB$ $BF := \frac{BM}{N_1}$

$EF := \sqrt{BE^2 - BF^2}$ $EI := 2 \cdot EF$

$EG := \frac{EI}{N_2}$ $FG := EG - EF$

$BG := \sqrt{BF^2 + FG^2}$ $GL := BL - BG$ $DG := GL$ $GH := \frac{FG \cdot GL}{BG}$ $HL := \sqrt{GL^2 - GH^2}$ $EH := EG + GH$

$EL := \sqrt{EH^2 + HI}$ $JL := \frac{EL}{2}$ $BJ := \sqrt{BL^2 - JL^2}$ $LN := \frac{BL \cdot JL}{BJ}$ $GN := \sqrt{LN^2 + GL^2}$ $JN := \sqrt{LN^2 - JL^2}$

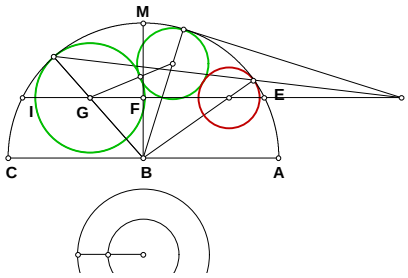
$EJ := JL$ $EN := \sqrt{JN^2 + EJ^2}$ $S_1 := EG$ $S_2 := EN$ $S_3 := GN$ $GO := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$NO := \sqrt{GN^2 - GO^2}$ $NR := \frac{NO^2}{GN}$ $GS := \frac{DG^2}{GN}$ $RS := GN - (NR + GS)$ $DT := RS$ $DS := \sqrt{DG^2 - GS^2}$

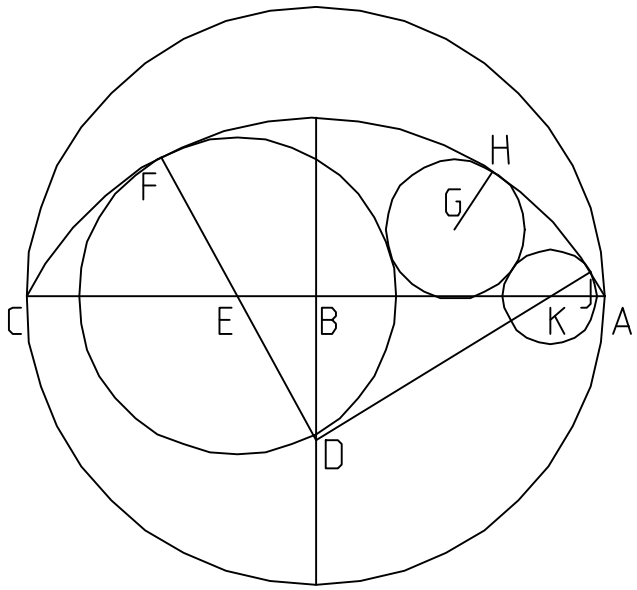
$RT := DS$ $OR := \sqrt{NO^2 - NR^2}$ $OT := OR - RT$ $DO := \sqrt{DT^2 + OT^2}$

$S_1 := GO$ $S_2 := DG$ $S_3 := DO$ $OQ := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$ $GQ := GO - OQ$ $DQ := \sqrt{DO^2 - OQ^2}$

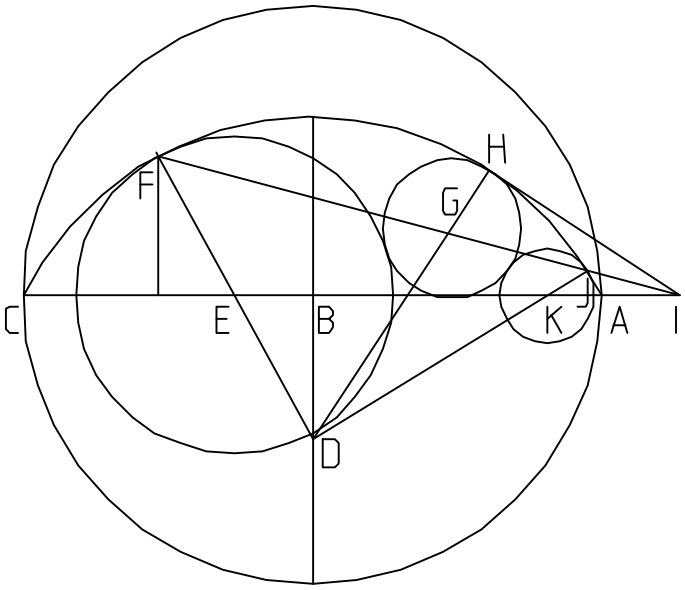
$FU := \frac{GQ \cdot BF}{DQ}$ $CE := \frac{BE \cdot EG}{FU + EF}$ $CE = 0.193$



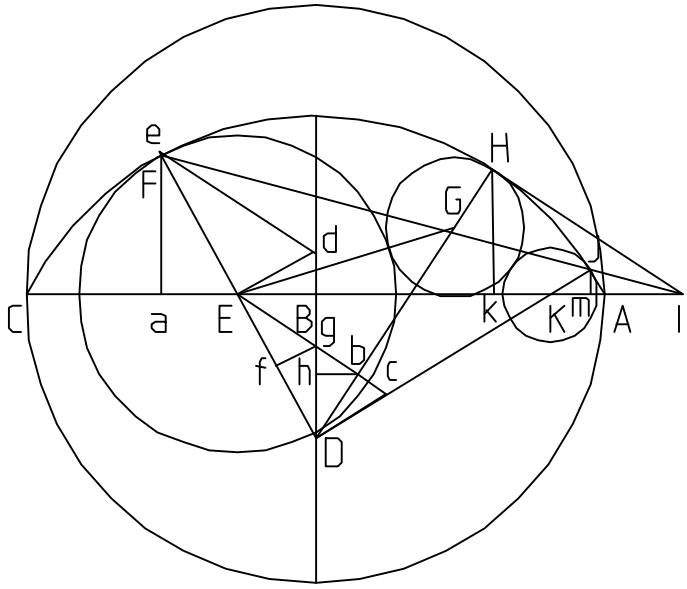
Edit In progress for three circles.



Place EF and GH and find JK.



Process summary



$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

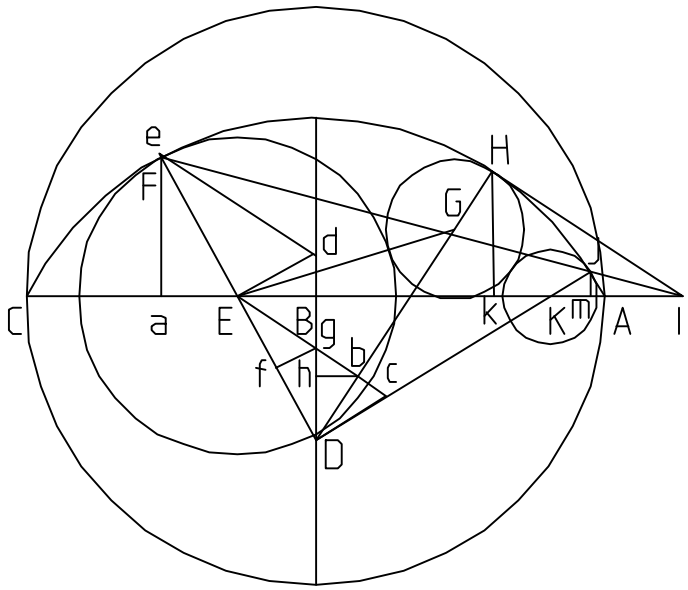
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$



$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

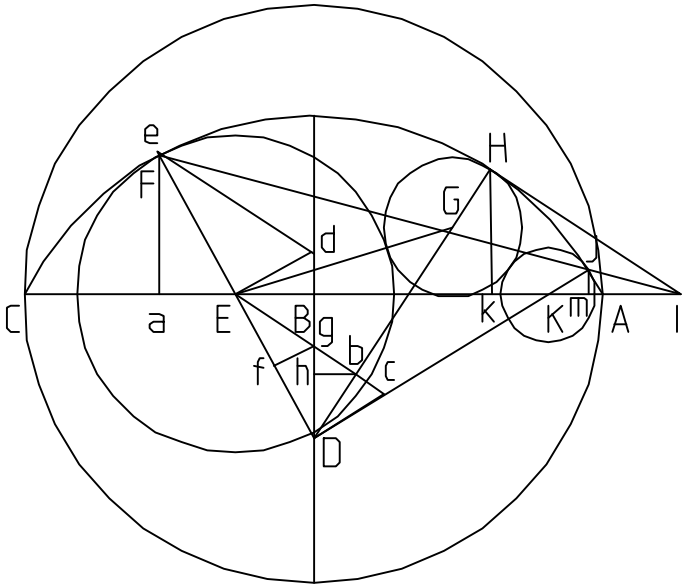
$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

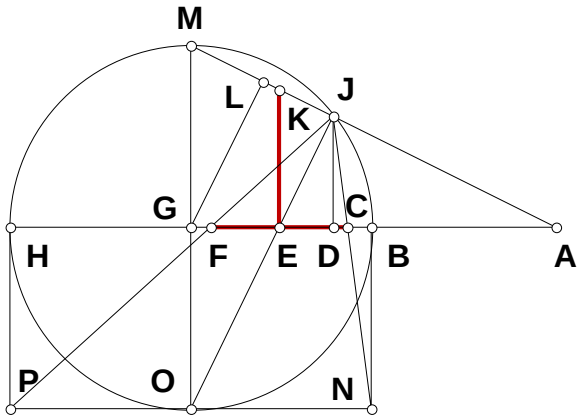
$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

$JK = 17.571$



When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.



Is CF always equal to EK?

$N_1 := 3$ $AB := 1$

$AH := AB \cdot N_1$ $BH := AH - AB$

$BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

$GM := BG$ $GH := BG$ $AG := AH - GH$

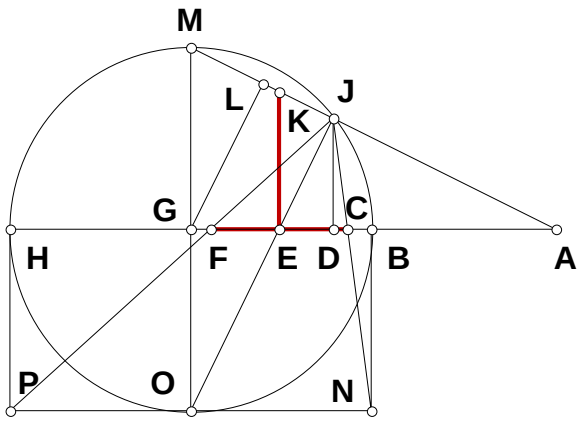
$AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$ $BD := AD - AB$

$DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$ $DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$

$CE := \frac{CF}{2}$ $BE := BC + CE$ $AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$

04/23/96 Plate II - Definition And Proof



Proof of equality in a tautologic is simply the demonstration that two names are synonyms. Is CF always equal to EK?

$N := 3 \qquad AB := 1 \qquad AH := N$

$BH := N - 1 \qquad BG := \frac{N - 1}{2}$

$AG := \frac{1}{2} \cdot N + \frac{1}{2} \qquad AM := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$AL := \frac{1}{2} \cdot \frac{(N + 1)^2}{\sqrt{2 \cdot N^2 + 2}} \qquad LM := \frac{1}{2} \cdot \frac{(N - 1)^2}{\sqrt{2 \cdot N^2 + 2}} \qquad AJ := \frac{2}{\sqrt{2 \cdot N^2 + 2}} \cdot N \qquad AD := (N + 1) \cdot \frac{N}{(N^2 + 1)}$

$BD := \frac{(N - 1)}{(N^2 + 1)} \qquad DH := N^2 \cdot \frac{(N - 1)}{(N^2 + 1)} \qquad DJ := \frac{(N - 1)}{(N^2 + 1)} \cdot N \qquad BC := \frac{(N - 1)}{(N + 1)^2}$

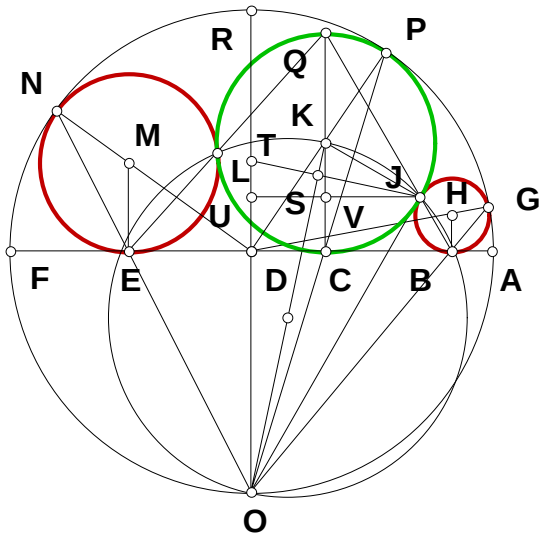
$DF := 2 \cdot N^3 \cdot \frac{(N - 1)}{[(N + 1)^2 \cdot (N^2 + 1)]} \qquad CD := 2 \cdot (N - 1) \cdot \frac{N}{[(N^2 + 1) \cdot (N + 1)^2]}$

$CF := 2 \cdot N \cdot \frac{(N - 1)}{(N + 1)^2} \qquad CE := N \cdot \frac{(N - 1)}{(N + 1)^2} \qquad BE := \frac{(N - 1)}{(N + 1)} \qquad AE := 2 \cdot \frac{N}{(N + 1)}$

$EK := 2 \cdot (N - 1) \cdot \frac{N}{(N + 1)^2} \qquad EK - CF = 0 \qquad EK = 0.75$

Meditation : Do these equations satisfy the requirement that a definition must contain both form and matter?

Three Circles 04/24/96



Given AC, find CK and BH.

$N := 1.5$ $AF := 1$ $AD := \frac{AF}{2}$

$AC := \frac{AF}{N}$ $DO := AD$ $OR := AF$

$CD := AD - AC$ $CO := \sqrt{CD^2 + DO^2}$

$PO := \frac{DO \cdot OR}{CO}$ $CP := PO - CO$ $CK := \frac{DO \cdot CP}{PO}$

$JK := CK$ $KO := \sqrt{CD^2 + (DO + CK)^2}$

$JO := \sqrt{KO^2 - JK^2}$ $KS := \frac{JK^2}{KO}$ $SO := KO - KS$ $JS := \frac{JK \cdot SO}{JO}$ $ST := \frac{CD \cdot SO}{DO + CK}$ $JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD}$ $TU := \frac{CD \cdot JT}{KO}$ $DU := TO - (DO + TU)$ $CV := DU$ $CQ := 2 \cdot CK$ $QV := CQ - CV$

$BH := \frac{CK \cdot CV}{QV}$ $BH = 0.19$

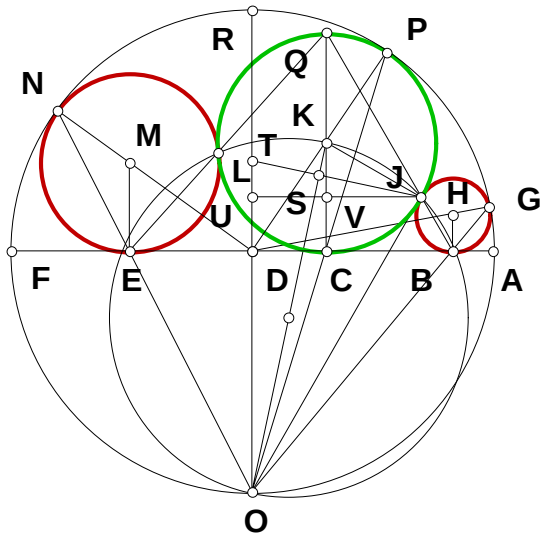
Some Algebraic Names

$AC := \frac{1}{N}$ $CD := \frac{1}{2} - \frac{1}{N}$ $CO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^2 - 2 \cdot N + 2}}{N}$ $PO := \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{N^2 - 2 \cdot N + 2}} \cdot N$

$CP := \sqrt{2} \cdot \frac{(N - 1)}{\left(\sqrt{N^2 - 2 \cdot N + 2} \cdot N\right)}$ $CK := \frac{(N - 1)}{N^2}$ $KO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}{N^2}$

$JO := \frac{1}{2} \cdot \sqrt{2}$ $KS := \frac{(N - 1)^2}{N^2} \cdot \frac{\sqrt{2}}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $SO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{N^2}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$

$JS := \frac{(N - 1)}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $ST := \frac{1}{2} \cdot (N - 2) \cdot \sqrt{2} \cdot \frac{N^3}{\left[\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2} \cdot \left(N^2 + 2 \cdot N - 2\right)\right]}$

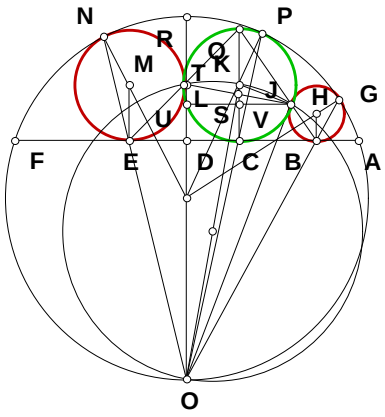


$$JT := \frac{1}{\left(2\cdot\sqrt{N^4+2\cdot N^2-4\cdot N+2}\right)}\cdot\frac{\left(2\cdot N^3+2\cdot N^2-8\cdot N+4+\sqrt{2}\cdot N^4-2\cdot\sqrt{2}\cdot N^3\right)}{\left(N^2+2\cdot N-2\right)} \qquad TO := \frac{N^2}{\left(N^2+2\cdot N-2\right)}$$

$$TU := \frac{1}{2}\cdot\left(N-\sqrt{2}\right)\cdot(N-2)\cdot\left(N-2+\sqrt{2}\right)\cdot\frac{N}{\left[\left(N^2+2\cdot N-2\right)\cdot\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)\right]}$$

$$DU := \frac{-1}{2}\cdot\left(\sqrt{2}-2\right)\cdot\frac{(N-1)}{\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)} \qquad CQ := 2\cdot\frac{(N-1)}{N^2}$$

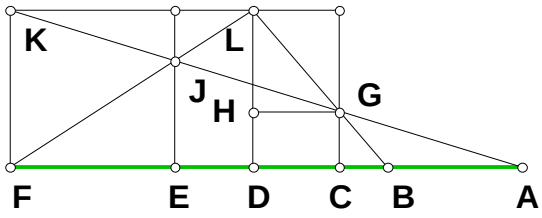
$$QV := \frac{1}{2}\cdot\left(\sqrt{2}+2\right)\cdot(N-1)\cdot\frac{\left(N+2-2\cdot\sqrt{2}\right)^2}{\left[\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)\cdot N^2\right]} \qquad BH := \frac{(N-1)\cdot\left(3-2\cdot\sqrt{2}\right)}{\left[(N+2)-2\cdot\sqrt{2}\right]^2}$$



Working on Moving D.

One Over N + One 04/25/96

Construct 1/(N+1)



$N_1 := 2.817 \quad N_2 := 3 \quad AC := 1$

$AF := AC \cdot N_1 \quad CF := AF - AC \quad CE := \frac{CF}{2}$

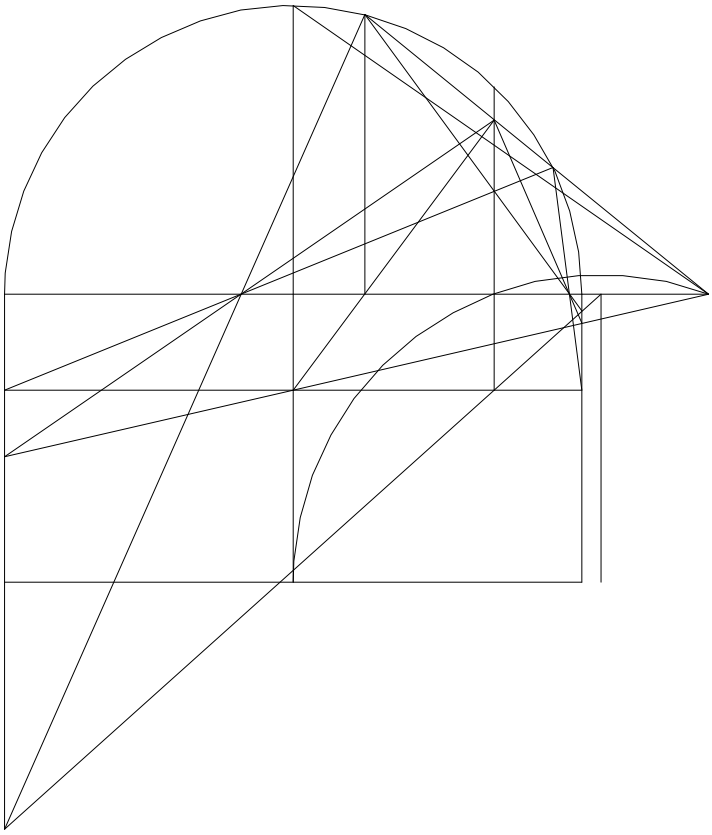
$AE := AC + CE \quad FK := AC \cdot N_2 \quad EJ := \frac{FK \cdot AE}{AF}$

$DL := FK \quad EF := CE \quad DF := \frac{EF \cdot DL}{EJ} \quad CG := \frac{FK \cdot AC}{AF} \quad CD := CF - DF \quad DH := CG \quad HL := DL - DH$

$BC := \frac{CD \cdot CG}{HL}$

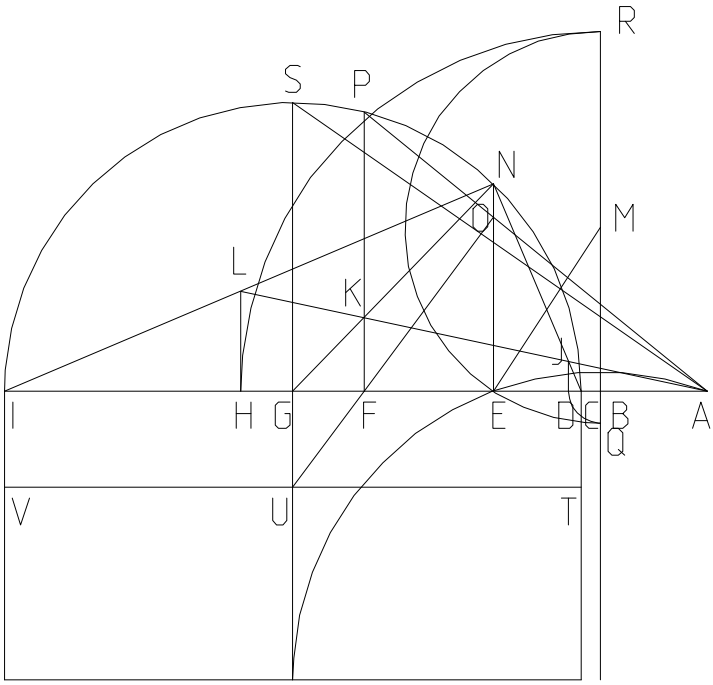
$CF := N_1 - 1 \quad CE := \frac{1}{2} \cdot (N_1 - 1) \quad AE := \frac{1}{2} \cdot (1 + N_1) \quad FK := N_2 \quad EJ := \frac{1}{2} \cdot N_2 \cdot \frac{(1 + N_1)}{N_1}$

$DF := \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 \quad CG := \frac{N_2}{N_1} \quad CD := \frac{(N_1 - 1)}{(1 + N_1)} \quad HL := N_2 \cdot \frac{(N_1 - 1)}{N_1} \quad \frac{1}{N_1 + 1} - BC = 0$



Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.



$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

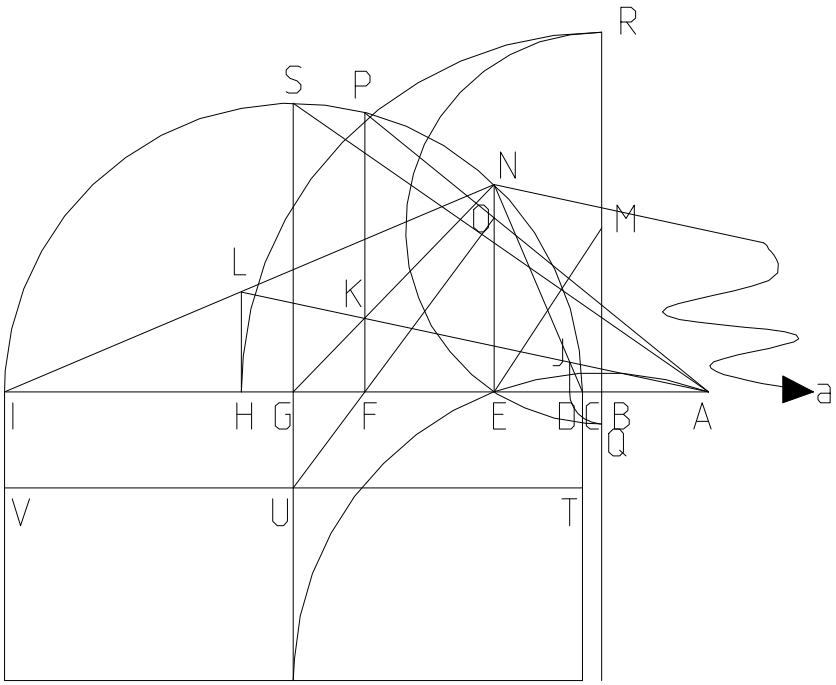
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



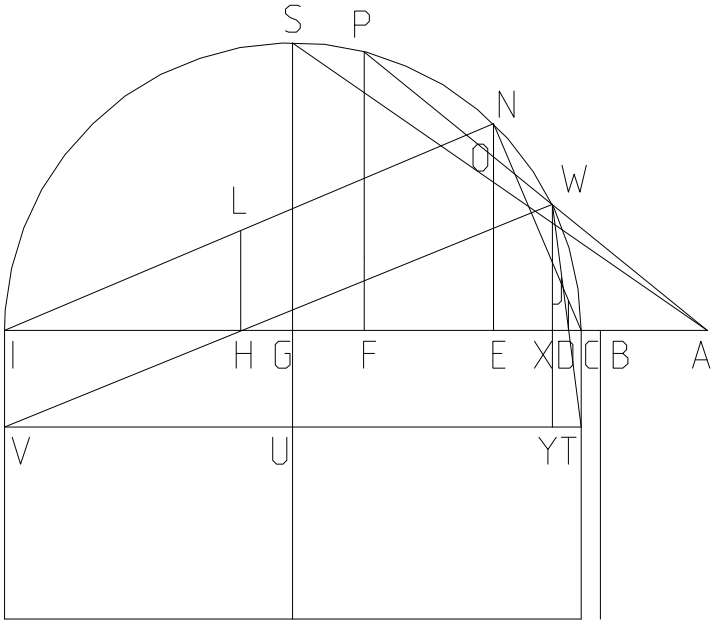
$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$



$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

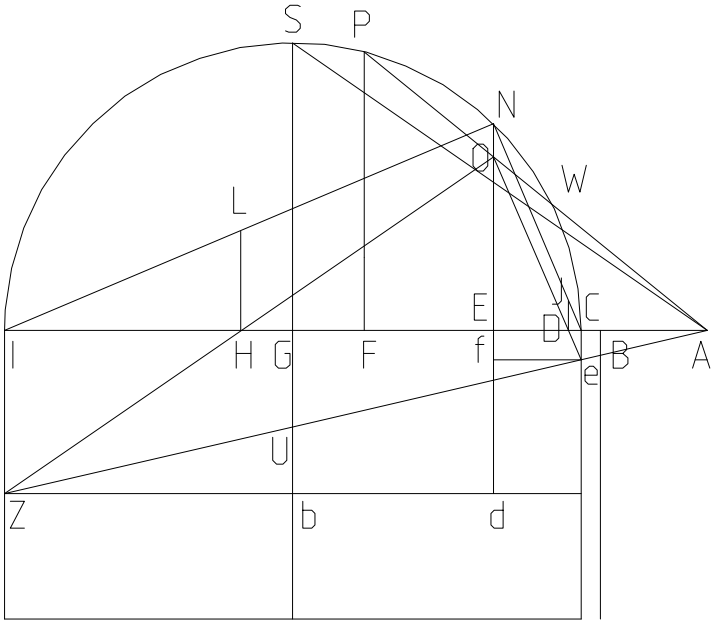
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

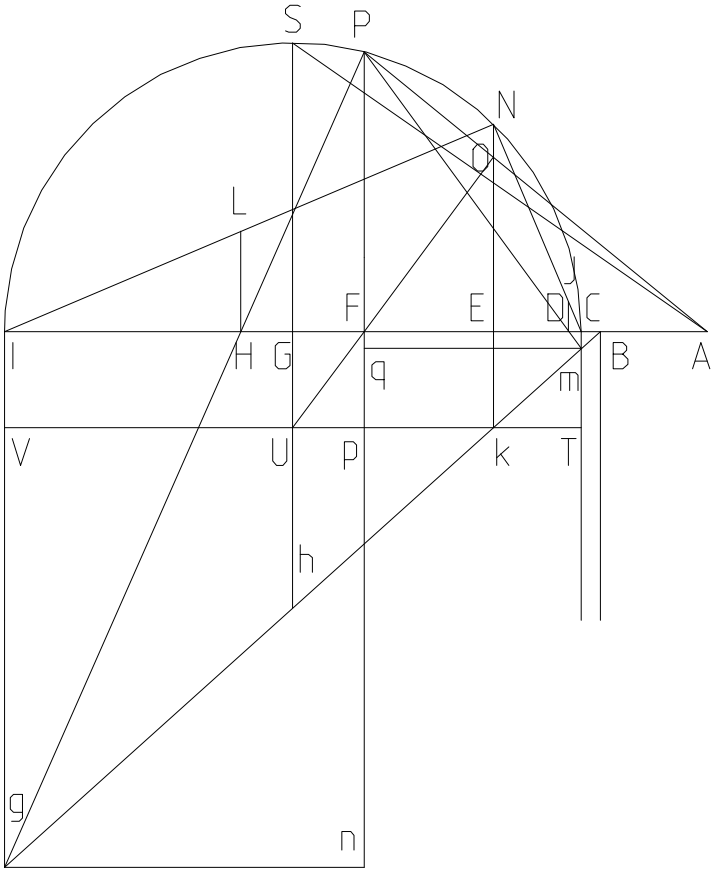
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$



$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

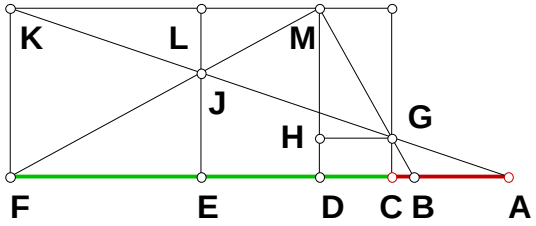
$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

A Root FIgure 04/27/96

Given CD, CE, and EF = CE, and that CD is the square root of BCxBF, find BC.



$N_1 := 5 \qquad N_2 := 3 \qquad CD := 1$

$CE := CD \cdot N_1 \qquad CF := 2 \cdot CE \qquad FK := CD \cdot N_2$

$DM := FK \qquad EL := FK \qquad DF := CF - CD$

$EF := \frac{CF}{2} \qquad EJ := \frac{DM \cdot EF}{DF} \qquad JL := EL - EJ$

$KL := EF \qquad AF := \frac{KL \cdot FK}{JL} \qquad AC := AF - CF \qquad CG := \frac{FK \cdot AC}{AF} \qquad DH := CG \qquad HM := DM - DH \qquad BC := \frac{CD \cdot DH}{HM}$

$BF := BC + CF \qquad BD := BC + CD \qquad \sqrt{BC \cdot BF} - BD = 0$

$CF := 2 \cdot N_1 \qquad FK := N_2 \qquad DF := \left(2 \cdot N_1 - 1\right) \qquad EF := N_1 \qquad EJ := \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1}$

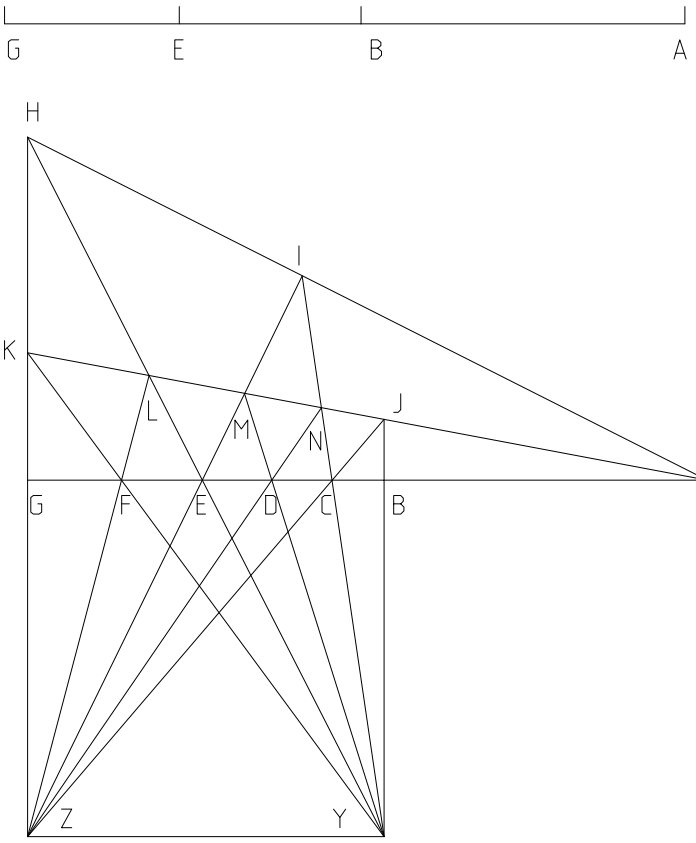
$JL := N_2 \cdot \frac{\left(N_1 - 1\right)}{\left(2 \cdot N_1 - 1\right)} \qquad AF := \frac{N_1 \cdot \left(2 \cdot N_1 - 1\right)}{N_1 - 1} \qquad AC := \frac{N_1}{\left(N_1 - 1\right)} \qquad CG := \frac{N_2}{\left(2 \cdot N_1 - 1\right)}$

$HM := 2 \cdot N_2 \cdot \frac{\left(N_1 - 1\right)}{\left(2 \cdot N_1 - 1\right)} \qquad BC := \frac{1}{2 \cdot \left(N_1 - 1\right)} \qquad BF := \frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 1\right)^2}{\left(N_1 - 1\right)} \qquad BD := \frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 1\right)}{\left(N_1 - 1\right)}$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use 5^{th} root series for example.



$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

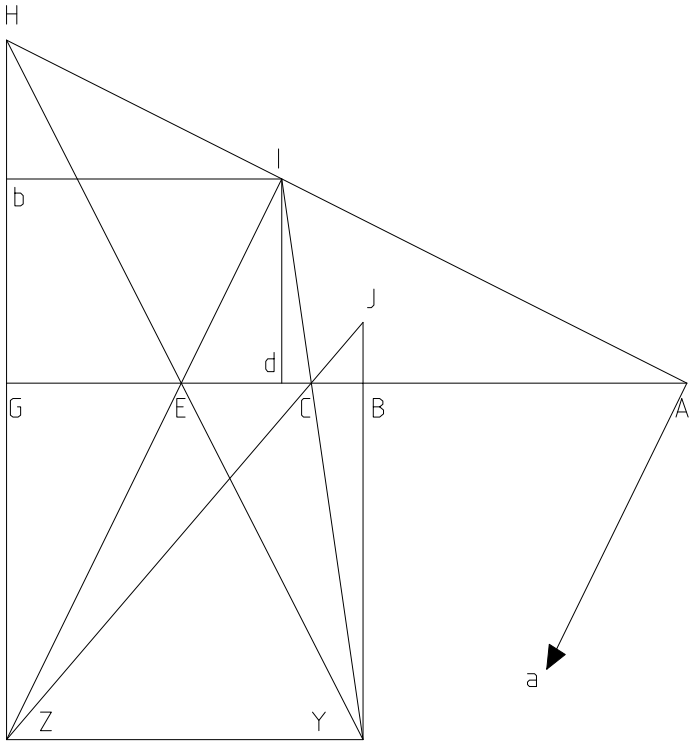
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

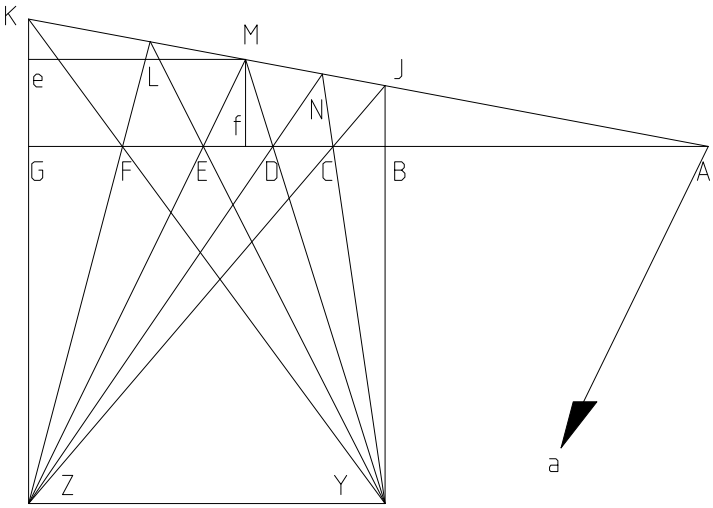
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



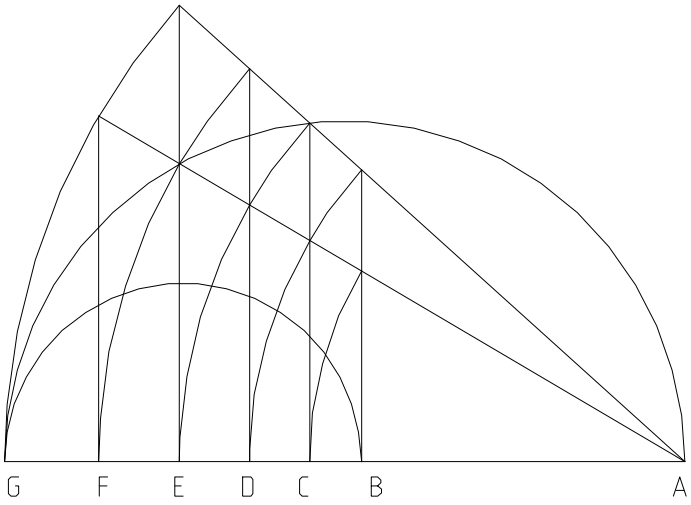


$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

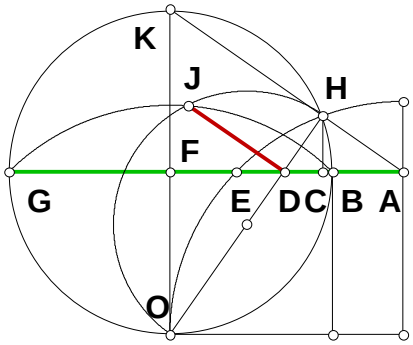
$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.

What is DJ? 04/29/96

DJ is the Geometric name, what is its Algebraic name?



$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N \qquad BG := AG - AB$

$BF := \frac{BG}{2} \qquad FK := BF \qquad FO := BF \qquad AF := BF + AB$

$DF := \frac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

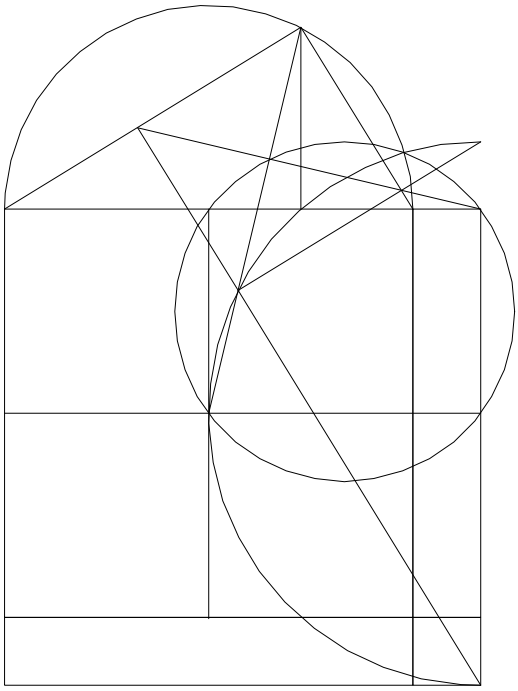
$HO := \frac{AF \cdot KO}{AK} \qquad DO := \frac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

$AG := N \qquad BG := N - 1 \qquad BF := \frac{N-1}{2} \qquad AF := \frac{1}{2} \cdot N + \frac{1}{2} \qquad DF := \frac{1}{2} \cdot \frac{(N-1)^2}{(N+1)} \qquad AK := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$HO := (N+1) \cdot \frac{(N-1)}{\sqrt{2 \cdot N^2 + 2}} \qquad DO := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2} \cdot \frac{(N-1)}{(N+1)} \qquad DH := 2 \cdot N \cdot \frac{(N-1)}{\left[\sqrt{2 \cdot N^2 + 2} \cdot (N+1)\right]}$

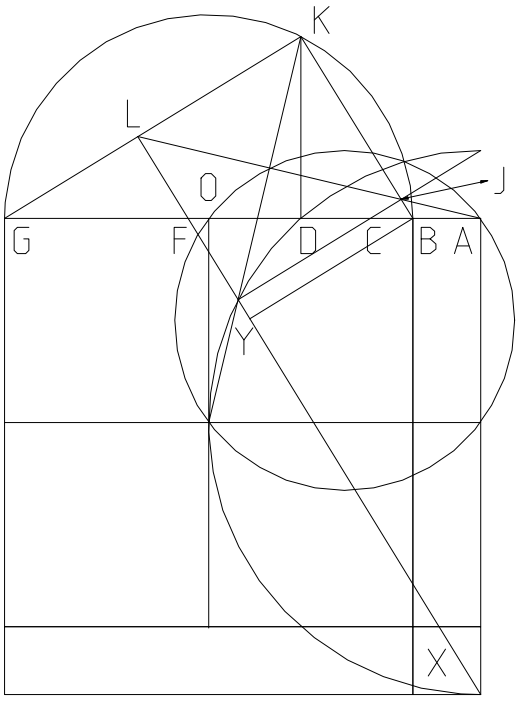
$$DJ := \sqrt{N} \cdot \frac{(N-1)}{(N+1)}$$



Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.



$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

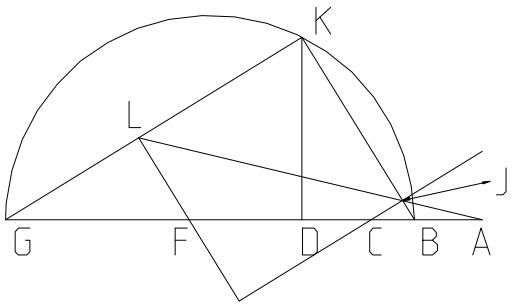
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



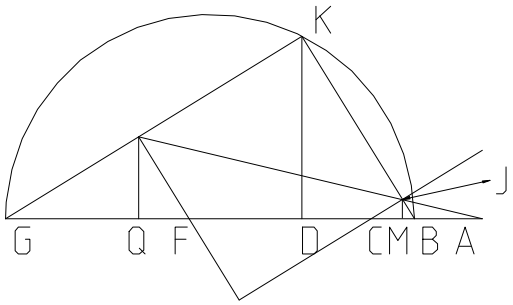
Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \quad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \quad \frac{BK}{BJ} = 8.075$$



$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

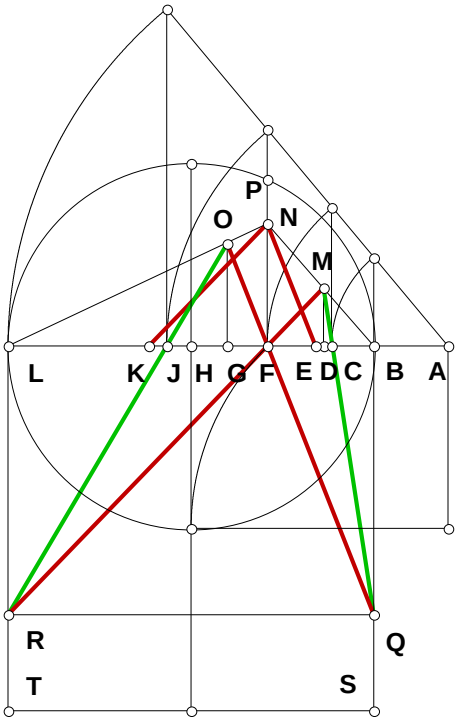
On the left is the first and last of the series, on the right is the entire series.

9.769
14.608
21.844
32.665

$\frac{AG}{BM} = 32.665$

12/20/96 Alternate Method Quad Roots

If FN:FP as BQ:BS then quad roots series can be divided off in the figure.



$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

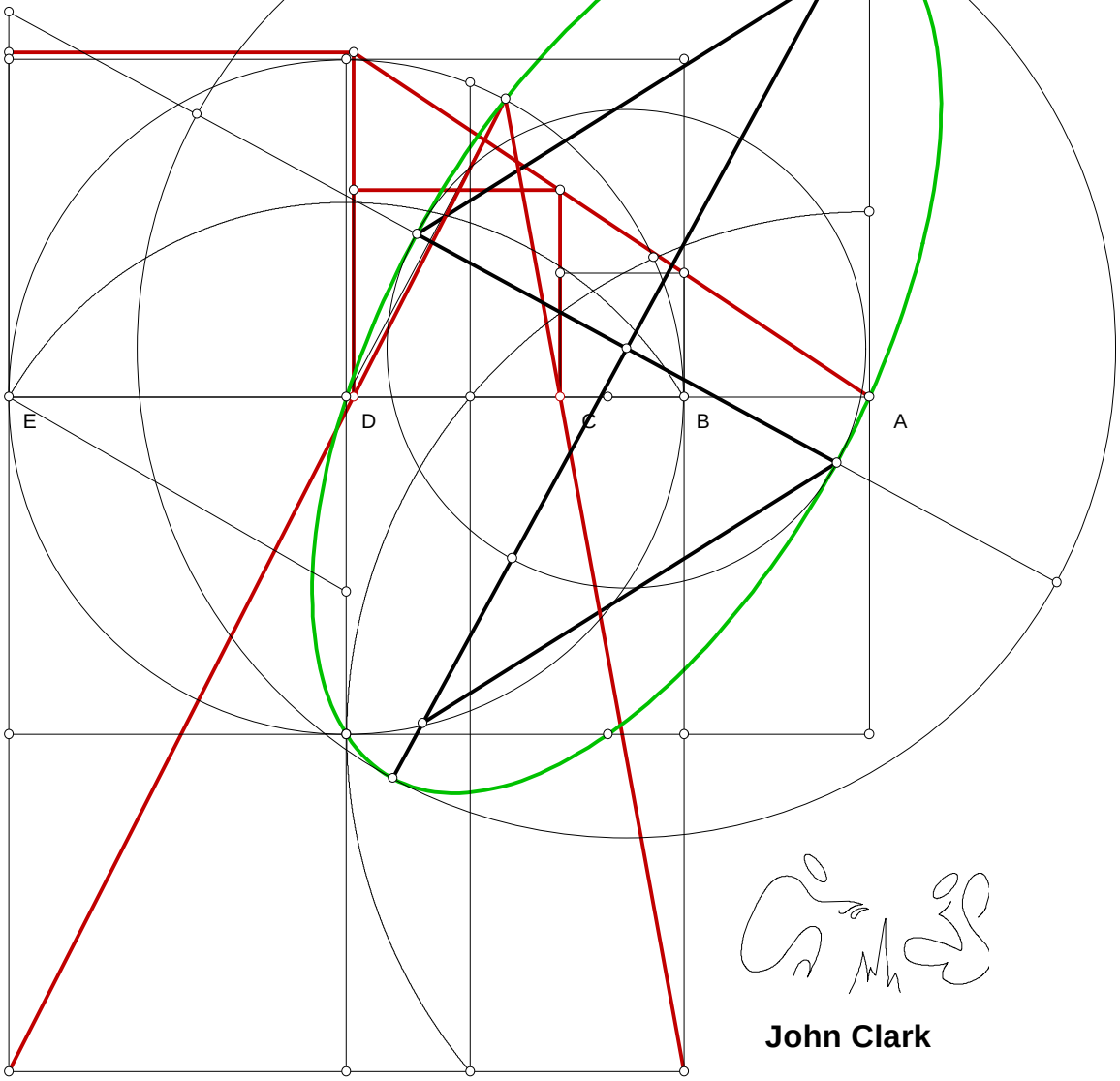
$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$

The Delian Quest

1997

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

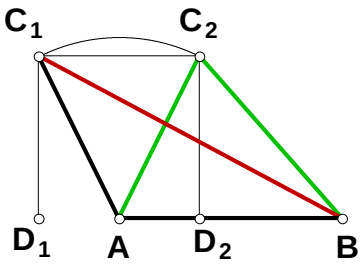
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

04/04/97 Triangles

Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.



$AB := 5 \qquad AC := 4 \qquad CD := 3$

$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$

$BC_1 := \sqrt{CD^2 + BD_1^2} \qquad BC_2 := \sqrt{CD^2 + BD_2^2}$

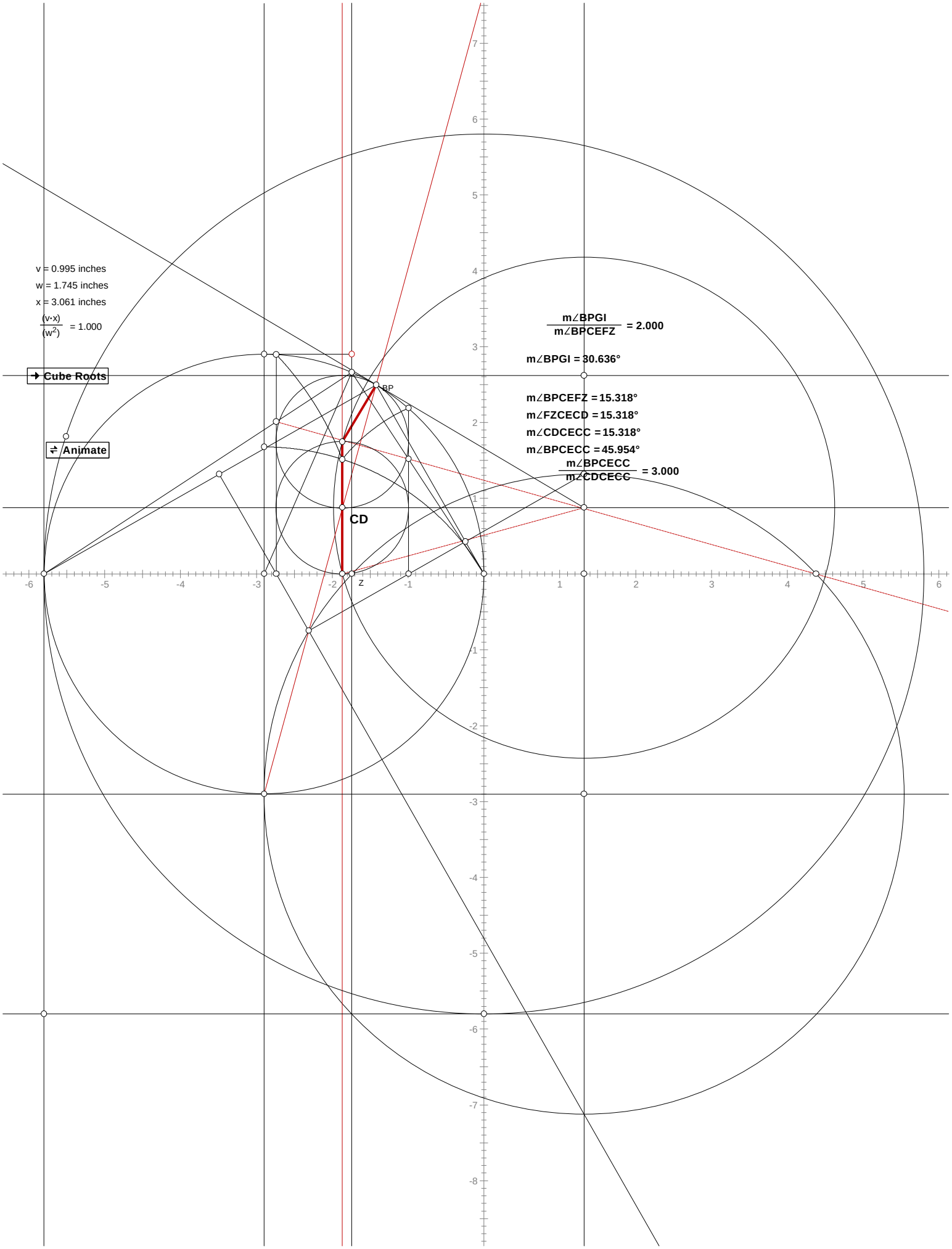
$BC_1 = 8.213 \qquad BC_2 = 3.813$

$S_1 := AB \quad S_2 := AC \quad S_3 := BC_1$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

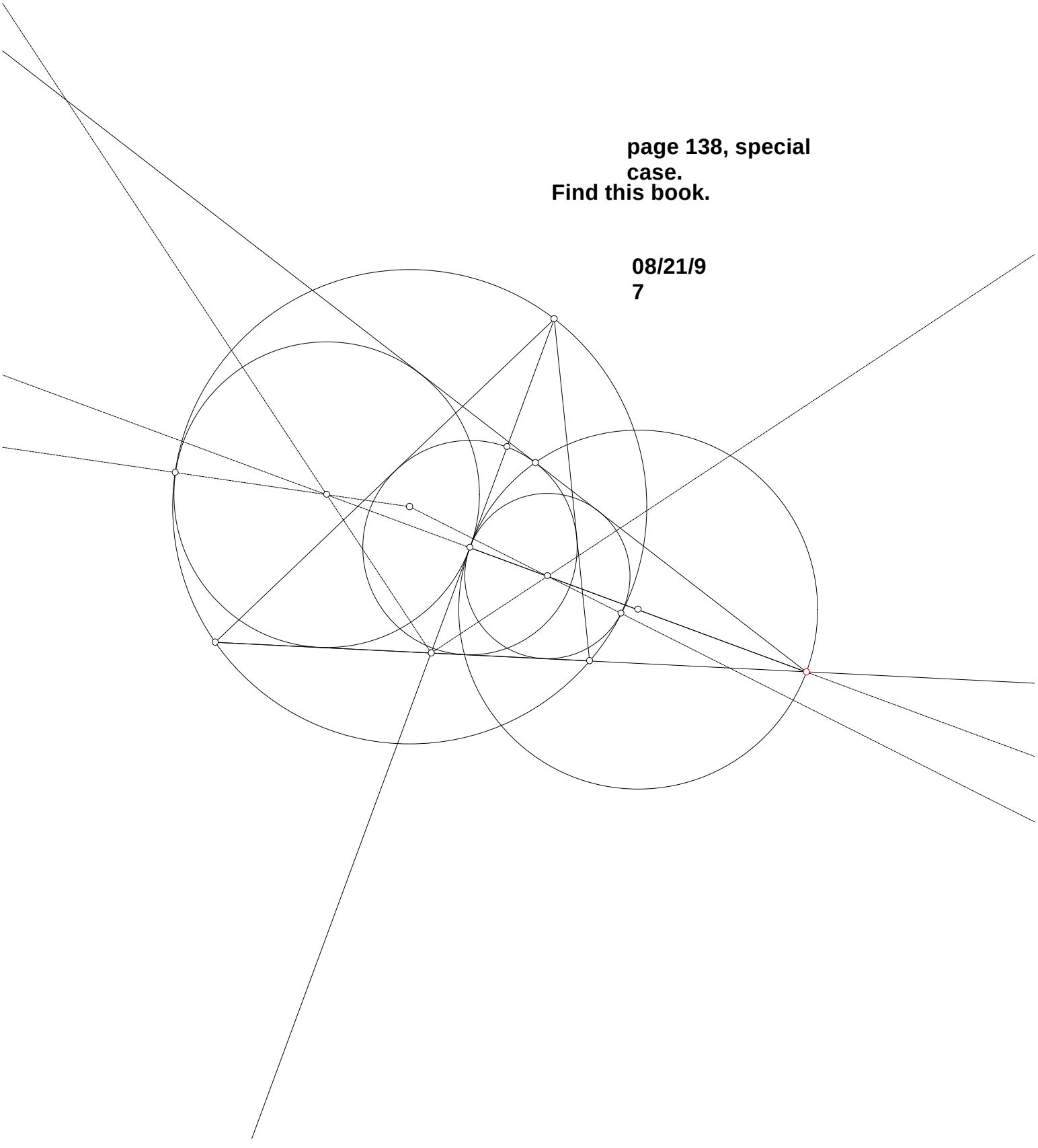
$S_1 := AB \quad S_2 := AC \quad S_3 := BC_2$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$


v = 0.995 inches
w = 1.745 inches
x = 3.061 inches
(v·x)/(w²) = 1.000
Cube Roots
Animate
m∠BPGI / m∠BPCEFZ = 2.000
m∠BPGI = 30.636°
m∠BPCEFZ = 15.318°
m∠FZCECD = 15.318°
m∠CDCECC = 15.318°
m∠BPCECC = 45.954°
m∠BPCECC / m∠CDCECC = 3.000
BP
CD
Z

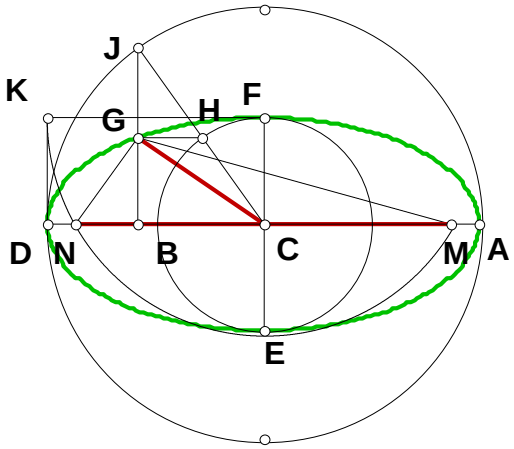
page 138, special case.
Find this book.

08/21/97



09/11/97 The Ellipse

Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.



$N_1 := 3 \qquad N_2 := 4$

$AD := 1 \qquad EF := \frac{AD}{N_1} \qquad AB := \frac{AD}{N_2}$

$BD := AD - AB \qquad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \qquad BC := AC - AB \qquad CH := \frac{EF}{2}$

$CJ := AC \qquad BG := \frac{BJ \cdot CH}{CJ}$

$CG := \sqrt{BG^2 + BC^2} \qquad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$\frac{\sqrt{4 \cdot N_2 - 4 + N_2^2 \cdot N_1^2 - 4 \cdot N_2 \cdot N_1^2 + 4 \cdot N_1^2}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{\sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0$$

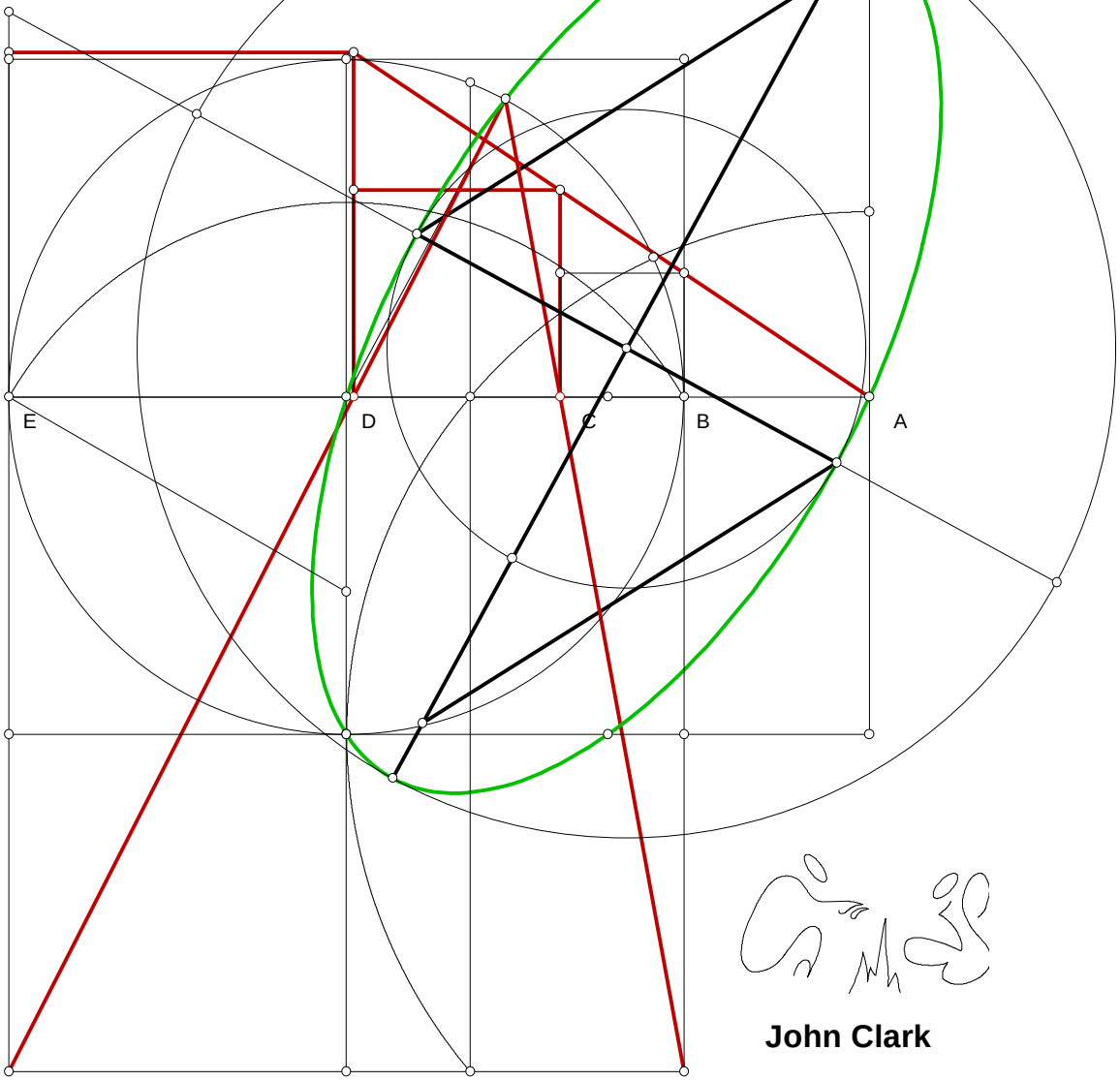
$$\frac{\sqrt{\left(N_1^2 - 1\right)}}{N_1} - MN = 0$$

The Delian Quest

1998

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

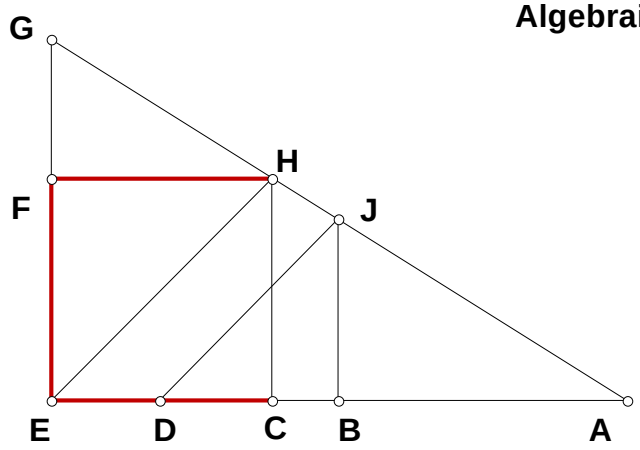
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

A Square In A Triangle 02/10/98

What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?



$N := 4 \quad AE := 1 \quad EG := N$

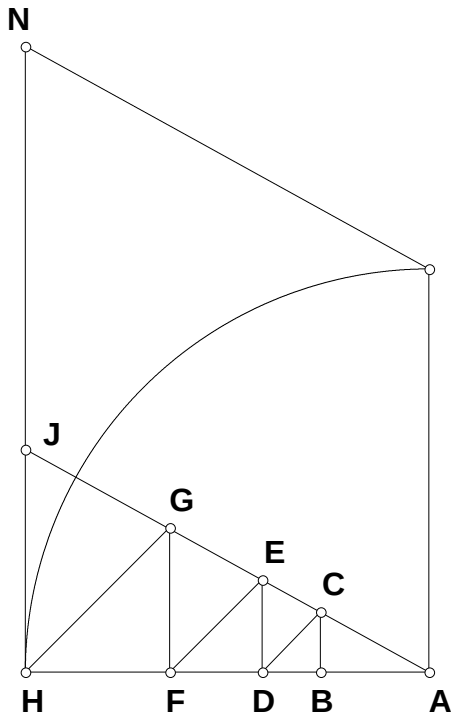
$AB := \frac{AE}{2} \quad BJ := \frac{EG}{2} \quad BD := BJ$

$AD := AB + BD \quad CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE \quad FG := EG - CE$

$\frac{AE}{AC} - (N + 1) = 0 \quad AC - \frac{1}{N + 1} = 0 \quad CE - \frac{N}{N + 1} = 0 \quad FG - \frac{N^2}{N + 1} = 0$

Alternate Method Root Series 02/25/98



Given a length and a unit, raise that length to any whole power.

Given for the third power.

$N := 1.3 \quad AH := 1 \quad HN := AH \cdot N$

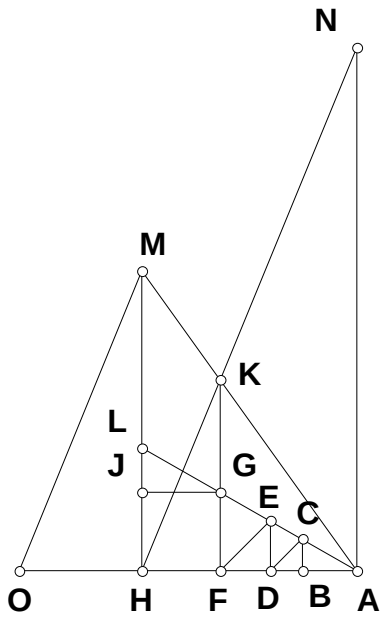
$HJ := HN - AH \quad FH := \frac{AH \cdot HJ}{AH + HJ} \quad AF := AH - FH$

$FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$\frac{AH}{AF} - N^1 = 0 \quad \frac{AH}{AD} - N^2 = 0 \quad \frac{AH}{AB} - N^3 = 0$

Sum DIvided by One Powered 02/25/98B



$N_1 := 5 \quad N_2 := 6 \quad AH := 1 \quad HM := AH \cdot N_1 \quad AN := AH \cdot N_2$

$HO := \frac{AH \cdot HM}{AN} \quad AO := AH + HO \quad AF := \frac{AH^2}{AO}$

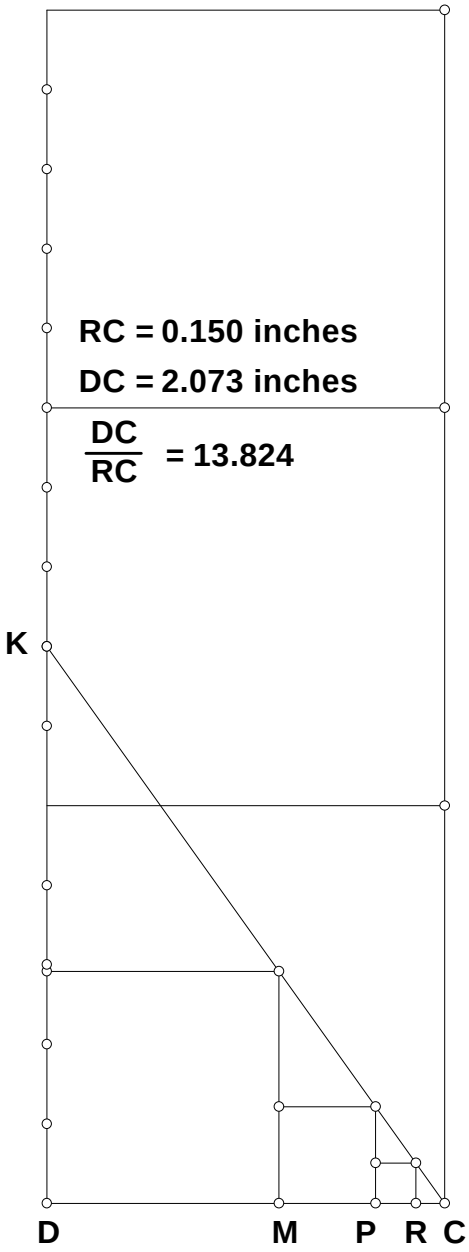
$FH := AH - AF \quad FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0$$

- ➜ 4
- ➜ 3+4/5
- ➜ 3+3/5
- ➜ 3+2/5
- ➜ 3+1/5
- ➜ 3
- ➜ 2+4/5
- ➜ 2+3/5
- ➜ 2+2/5
- ➜ 2+1/5
- ➜ 2
- ➜ 1+4/5
- ➜ 1+3/5
- ➜ 1+2/5
- ➜ 1+1/5
- ▲ Show
- △ Hide

M^N



$4^3 = 64.000$

$\left(3 + \frac{4}{5}\right)^3 = 54.872$

$\left(3 + \frac{3}{5}\right)^3 = 46.656$

$\left(3 + \frac{2}{5}\right)^3 = 39.304$

$\left(3 + \frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2 + \frac{4}{5}\right)^3 = 21.952$

$\left(2 + \frac{3}{5}\right)^3 = 17.576$

$\left(2 + \frac{2}{5}\right)^3 = 13.824$

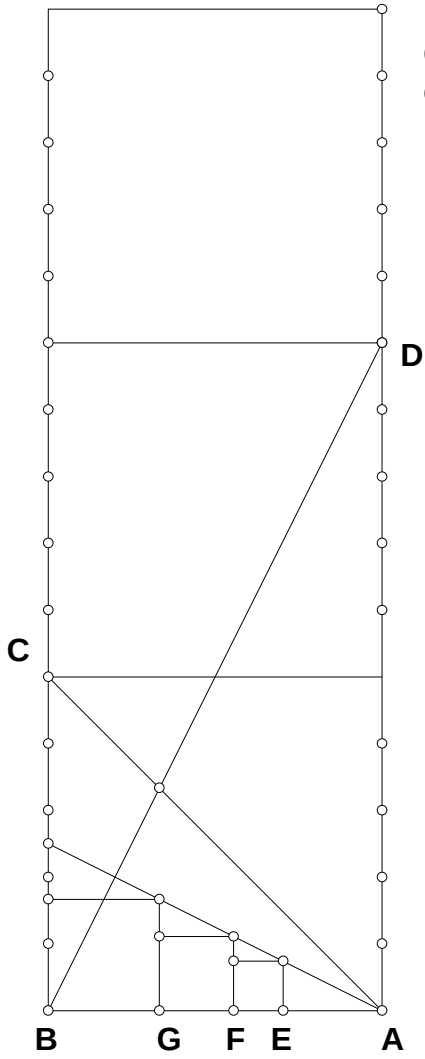
$\left(2 + \frac{1}{5}\right)^3 = 10.648$

$\left(1 + \frac{4}{5}\right)^3 = 5.832$

$\left(1 + \frac{3}{5}\right)^3 = 4.096$

$\left(1 + \frac{2}{5}\right)^3 = 2.744$

$\left(1 + \frac{1}{5}\right)^3 = 1.728$



Given line AB, divide it by the equation

$$\left(\frac{(BC + AD)}{AD} \right)^{W}$$

where W is a whole number.

BC

- 2+4/5
- 2+3/5
- 2+2/5
- 2+1/5
- 2
- 1+4/5
- 1+3/5
- 1+2/5
- 1+1/5
- 4
- 3+4/5
- 3+3/5
- 3+2/5
- 3+1/5
- 3

AD

- 2+4/5
- 2+3/5
- 2+2/5
- 2+1/5
- 2
- 1+4/5
- 1+3/5
- 1+2/5
- 1+1/5
- 4
- 3+4/5
- 3+3/5
- 3+2/5
- 3+1/5
- 3

$$\frac{AB}{AG} = 1.500$$

$$\frac{AB}{AF} = 2.250$$

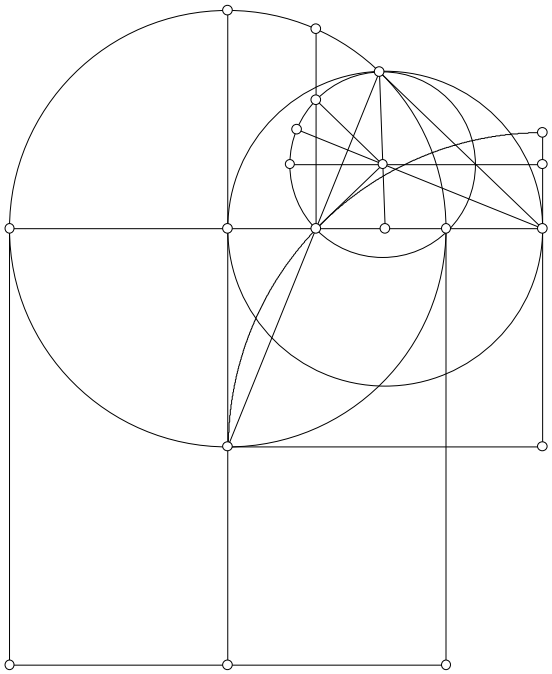
$$\frac{AB}{AE} = 3.375$$

$$\left(\frac{(BC + AD)}{AD} \right)^{1} = 1.500$$

$$\left(\frac{(BC + AD)}{AD} \right)^{2} = 2.250$$

$$\left(\frac{(BC + AD)}{AD} \right)^{3} = 3.375$$

042398



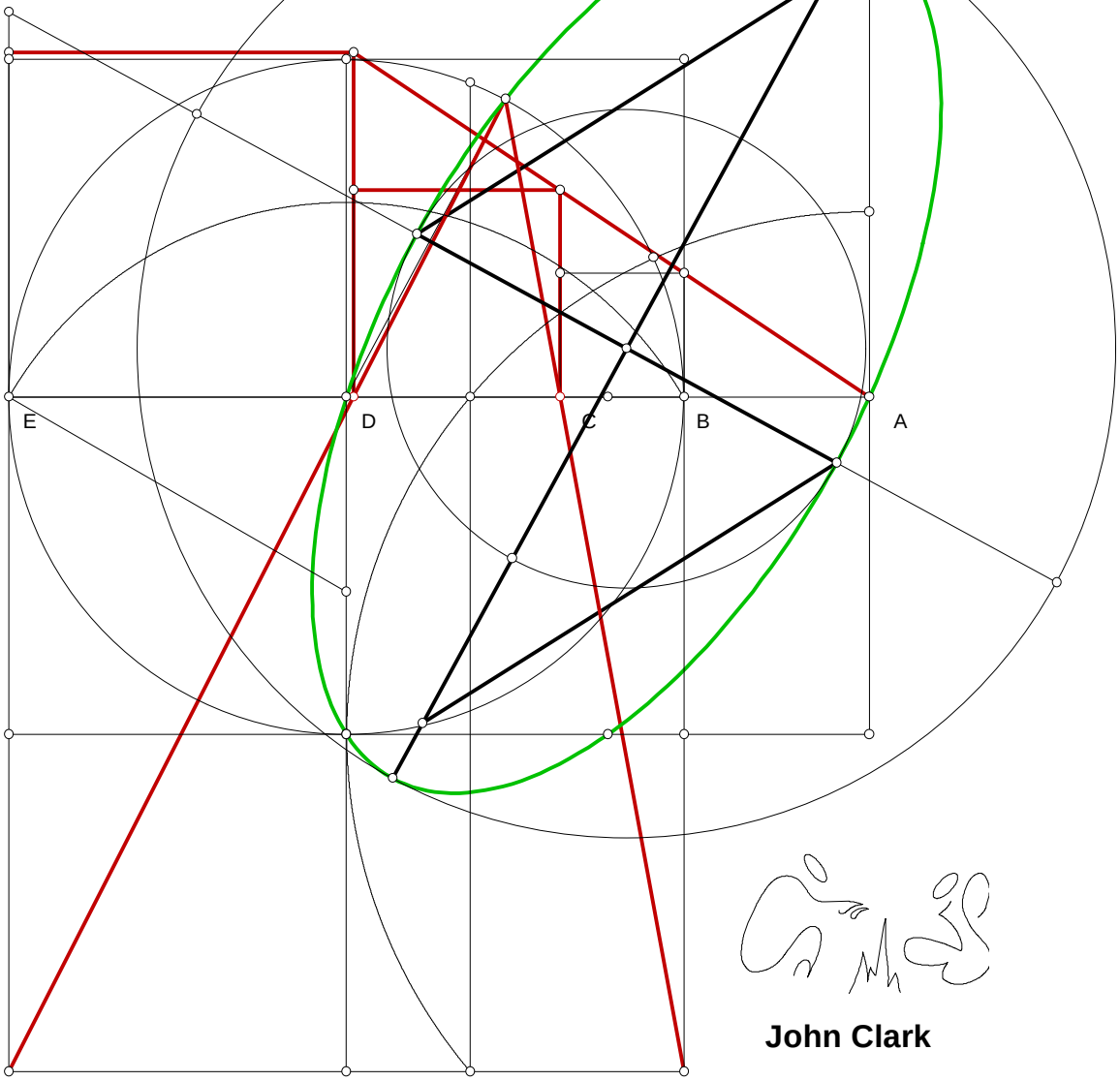
Trisection and Square Roots

The Delian Quest

1999

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

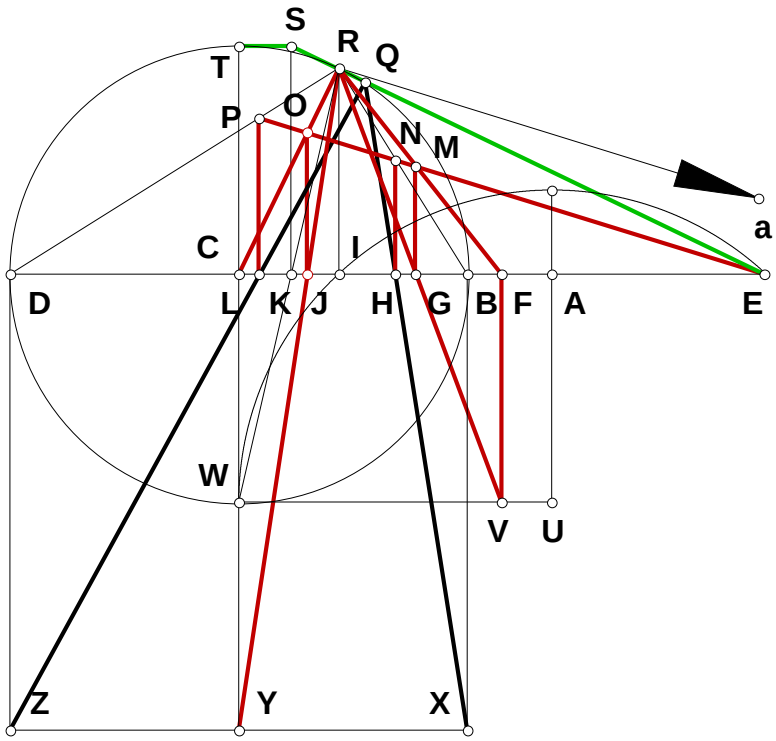
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



John Clark

On Gemini Roots 07/24/99

CE is to EF as CY is to CW



$N_1 := 6.429$ $N_2 := 2$

$AB := .438$ $AD := AB \cdot N_1$

$BD := AD - AB$ $BC := \frac{BD}{2}$

$CW := BC$ $CT := BC$

$FV := BC$ $AI := \sqrt{AB \cdot AD}$

$BI := AI - AB$ $DI := BD - BI$

$IR := \sqrt{BI \cdot DI}$ $AE := AI$

$AC := AB + BC$ $ED := AD + AE$

$CE := AC + AE$ $EF := \frac{CE}{N_2}$

$BE := AE + AB$ $EI := AE + AI$

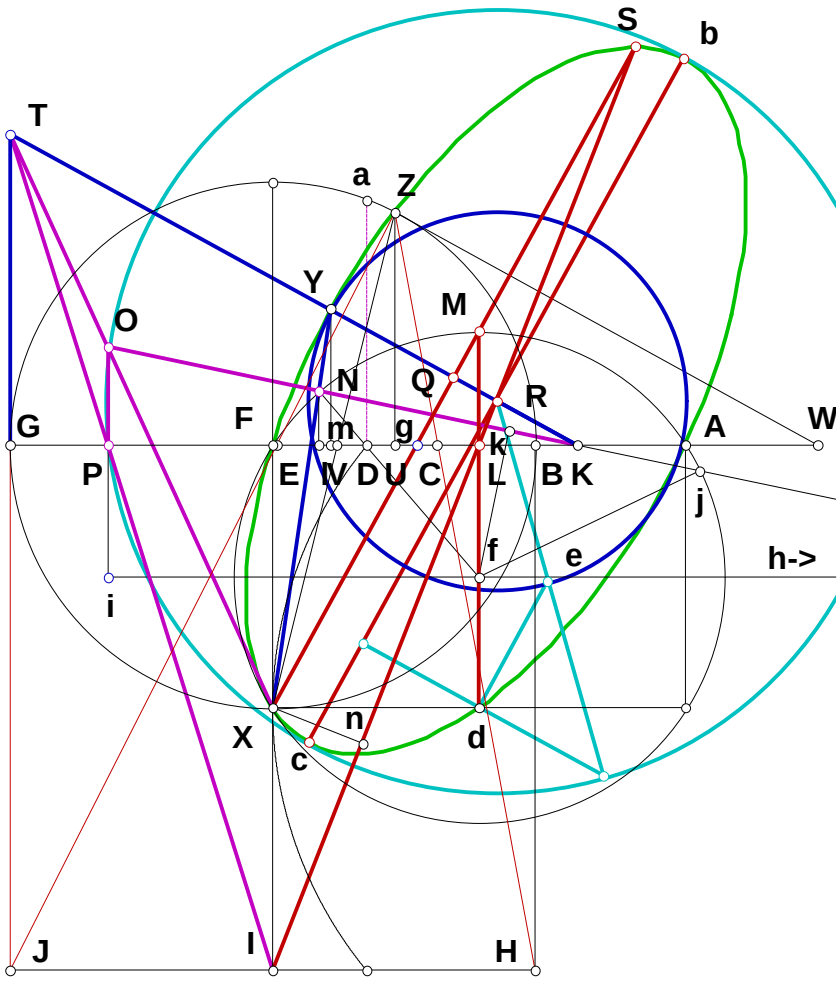
$FI := EI - EF$ $FG := \frac{FI \cdot FV}{FV + IR}$

$EG := EF + FG$ $GI := FI - FG$ $GM := \frac{FV \cdot GI}{FI}$ $Ia := \frac{EG \cdot IR}{GM}$ $EL := \frac{Ia \cdot ED}{Ia + DI}$ $BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI$ $BH := \frac{BI \cdot BE}{Ba}$ $EH := BE + BH$ $CI := AC - AI$ $JO := \frac{IR \cdot CE}{CI + Ia}$ $CJ := \frac{CI \cdot JO}{IR}$

$JI := CI - CJ$ $CY := \frac{IR \cdot CJ}{JI}$ $\frac{CY}{CW} - \frac{CE}{EF} = 0$

A Delian Solution 08/11/99



What are the minor and major axis for the ellipse that will give point Z for the cube root?

$AB := 1 \qquad N := 16 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad AD := \sqrt{AB \cdot AG}$

$BF := \frac{BG}{2} \qquad FG := BF \qquad AF := AB + BF$

$FX := BF \qquad Mf := \frac{\sqrt{AF^2 + FX^2}}{2} \qquad Lf := \frac{FX}{2}$

$ML := Mf - Lf \qquad FL := \frac{AF}{2} \qquad Xd := FL \qquad df := Lf$

$IX := FX \qquad Md := Mf + df \qquad MX := \sqrt{Xd^2 + Md^2}$

$SX := \frac{MX \cdot IX}{IX - ML} \qquad Lg := \frac{FL \cdot ML}{ML + FX} \qquad QX := \frac{SX}{2}$

$Fg := FL - Lg \qquad Xg := \frac{MX \cdot Fg}{Xd} \qquad Qg := QX - Xg$

$Kg := \frac{Xg \cdot Qg}{Fg} \qquad GK := FG + Fg + Kg \qquad GJ := BG$

$GT := \frac{Fg \cdot GK}{FX} \qquad JT := GJ + GT \qquad IJ := BF$

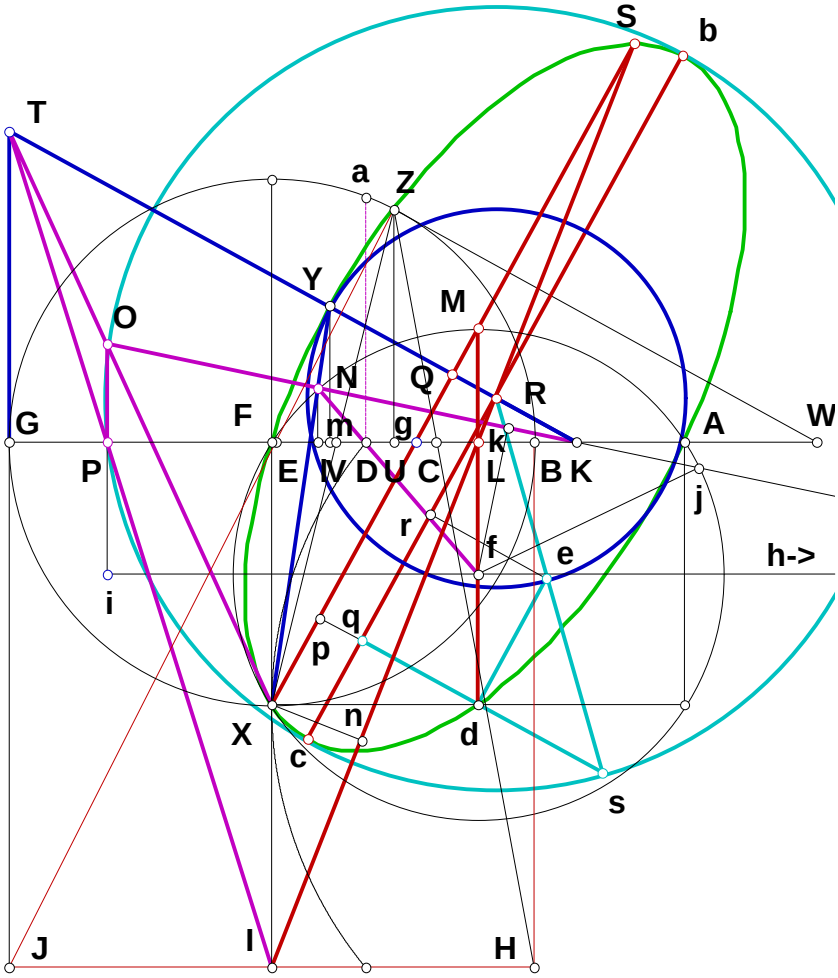
$FP := \frac{IJ \cdot GJ}{JT} \qquad OP := \frac{IX \cdot GT}{JT}$

$KP := Fg + Kg + FP \qquad Pi := Lf \qquad Oi := OP + Pi \qquad hi := \frac{KP \cdot Oi}{OP} \qquad fi := FP + FL \qquad fh := hi - fi \qquad KO := \sqrt{KP^2 + OP^2}$

$hk := \frac{KP \cdot fh}{KO} \qquad fk := \frac{OP \cdot fh}{KO} \qquad Nf := Mf \qquad Nk := \sqrt{Nf^2 - fk^2} \qquad Nh := hk + Nk \qquad Oh := \frac{KO \cdot Oi}{OP} \qquad NO := Oh - Nh \qquad KN := KO - NO$

$$Nl := \frac{OP \cdot KN}{KO} \quad Kl := \frac{KP \cdot KN}{KO} \quad FK := KP - FP \quad Fl := FK - Kl \quad NX := \sqrt{(FX + Nl)^2 + Fl^2} \quad XY := \frac{NX \cdot IX}{IX - Nl} \quad Fm := \frac{Fl \cdot XY}{NX}$$

$$Km := FK - Fm \quad Fo := \frac{Fl \cdot FX}{FX + Nl} \quad Xo := \frac{NX \cdot Fo}{Fl} \quad mo := Fm - Fo \quad Ym := \frac{FX \cdot mo}{Fo} \quad FI := 2 \cdot BF \quad IL := \sqrt{FL^2 + FI^2}$$



$$IS := \frac{IL \cdot IX}{IX - ML} \quad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

$$Xn := \sqrt{IX^2 - In^2} \quad QR := \frac{Xn \cdot QX}{IS - In}$$

$$KG := FK + FG \quad KT := \sqrt{KG^2 + GT^2}$$

$$KY := \frac{KT \cdot Ym}{GT} \quad KQ := \frac{KG \cdot Qg}{GT}$$

$$RY := KY - KQ + QR \qquad RY = 4.119$$

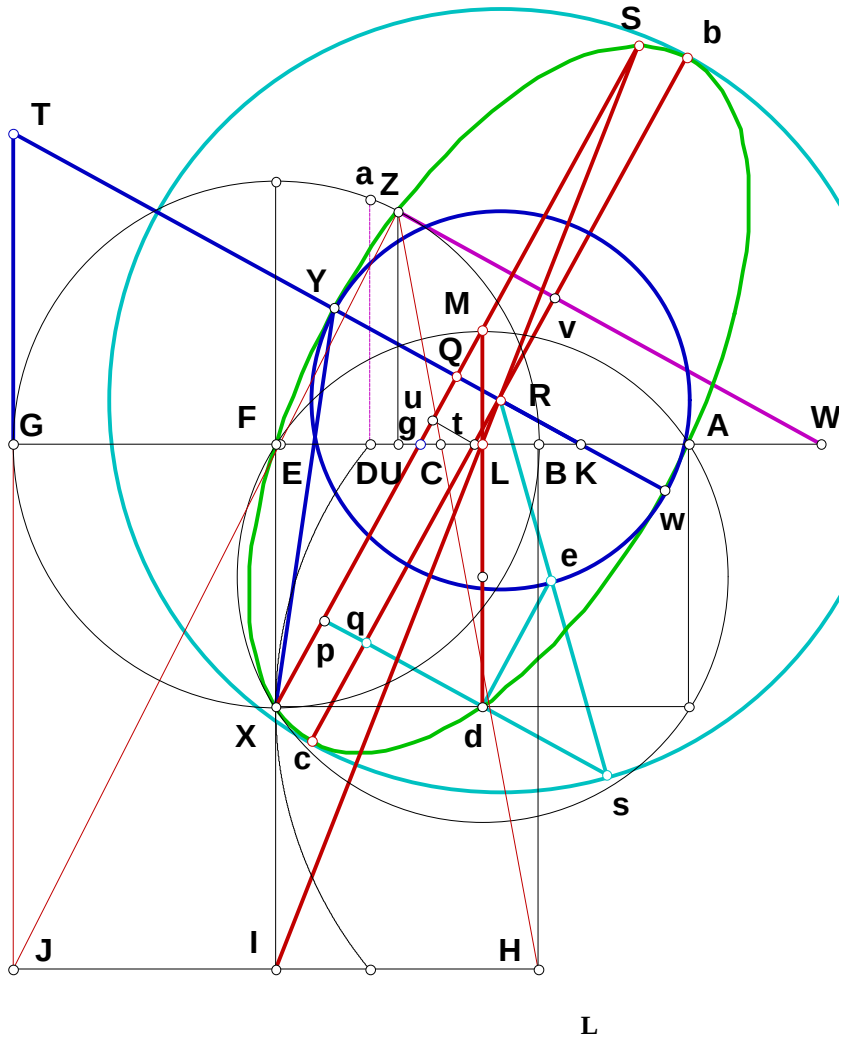
$$Xd := FL \quad dp := \frac{GK \cdot Xd}{KT} \quad pq := QR \quad dq := dp - pq$$

$$Xp := \frac{GT \cdot Xd}{KT} \quad Qp := QX - Xp \quad Rq := Qp$$

$$er := dq \quad Re := RY \quad Rr := \sqrt{Re^2 - er^2}$$

$$Rs := \frac{Re \cdot Rq}{Rr} \qquad Rs = 7.171$$

Is the segment Zv equal to the perpendicular for the ellipse?



$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \qquad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}}$$

$$BC := AC - AB \qquad BE := AE - AB$$

$$CE := BE - BC \qquad EG := BG - BE$$

$$CU := \frac{BC \cdot CE}{BC + EG} \qquad BU := BC + CU$$

$$GU := BG - BU \qquad UZ := \sqrt{BU \cdot GU}$$

$$UW := \frac{KG \cdot UZ}{GT} \qquad Gg := GK - Kg$$

$$tu := QR \qquad gt := \frac{KT \cdot tu}{GK} \qquad Gt := Gg + gt$$

$$GW := GU + UW \qquad Wt := GW - Gt$$

$$tv := \frac{GT \cdot Wt}{KT} \qquad Kt := GK - Gt \qquad Rt := \frac{GT \cdot Kt}{KT}$$

$$Rv := tv - Rt \qquad bc := 2 \cdot Rs \qquad Rc := Rs$$

$$cv := Rc + Rv \qquad Yw := 2 \cdot RY \qquad WZ := \frac{KT \cdot UZ}{GT}$$

$$Wv := \frac{GK \cdot tv}{GT} \qquad Zv := \left| WZ - Wv \right|$$

$N_1 := Yw \qquad N_2 := bc \qquad N_3 := cv \qquad N_4 := bc \qquad \sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2} - Zv = 0 \qquad \frac{AC}{2 \cdot AB} - 2^{\frac{1}{3}} = 0$

$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

PD = 0.601 inches

PC = 5.299 inches

$\frac{PC}{PD}$ = 8.821

ARAS = 1.323 inches

QR = 1.323 inches

ARAS - QR = 0.000 inches

PD = 0.601 inches

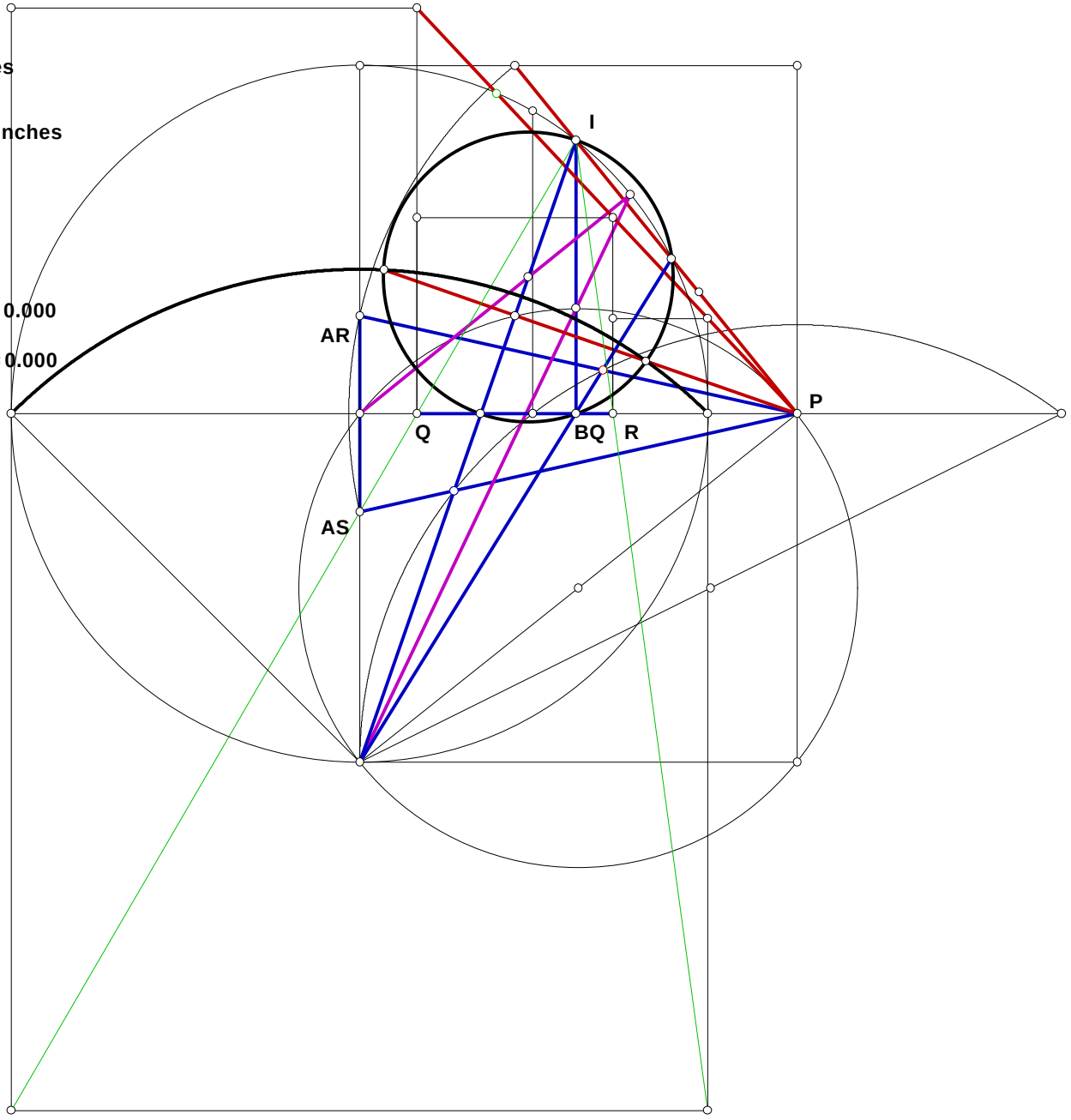
PC = 5.299 inches

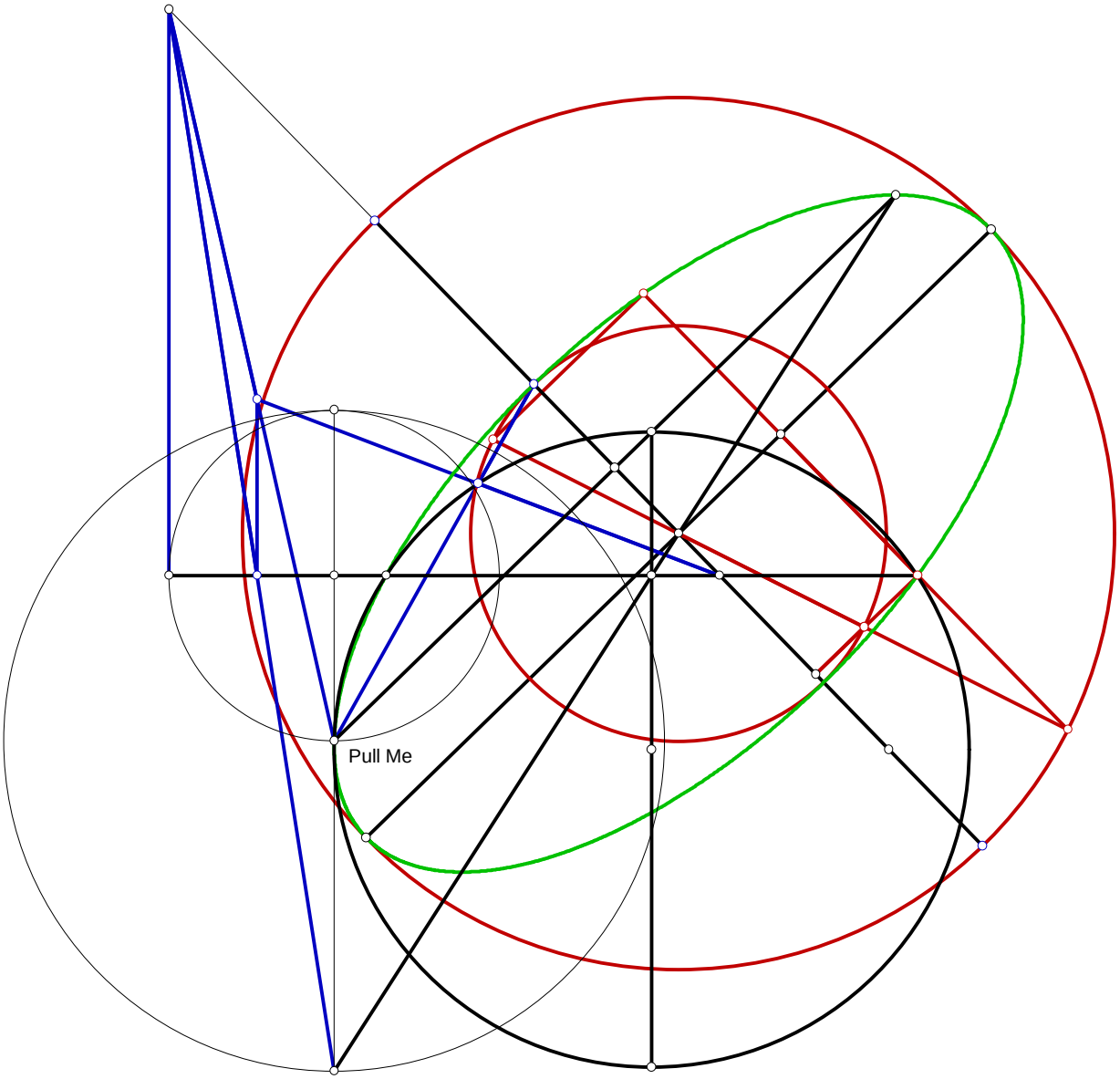
PR = 1.241 inches

PQ = 2.564 inches

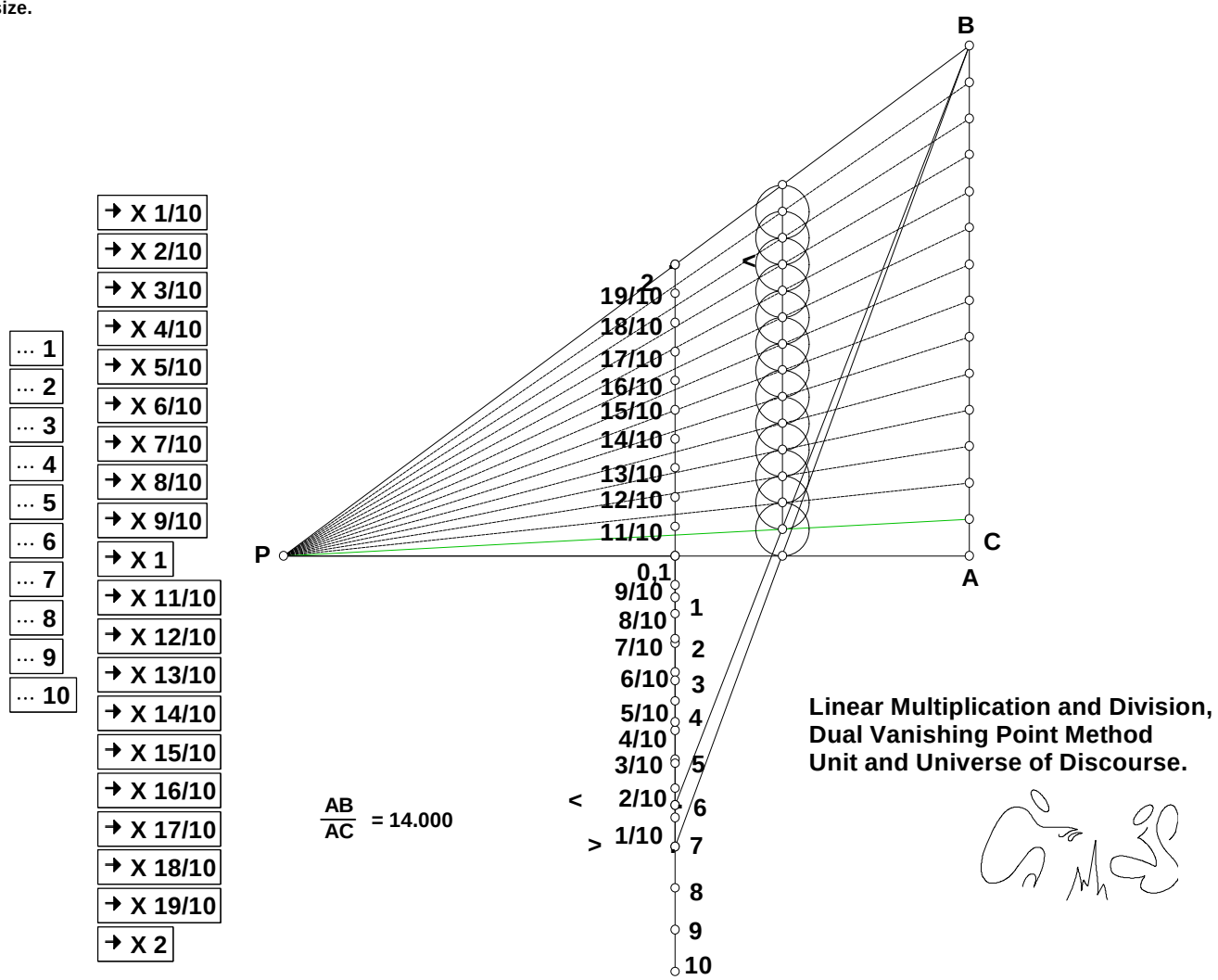
$(PD^2 \cdot PC)\left(\frac{1}{3}\right)$ - PR = 0.000

$(PD \cdot PC^2)\left(\frac{1}{3}\right)$ - PQ = 0.000



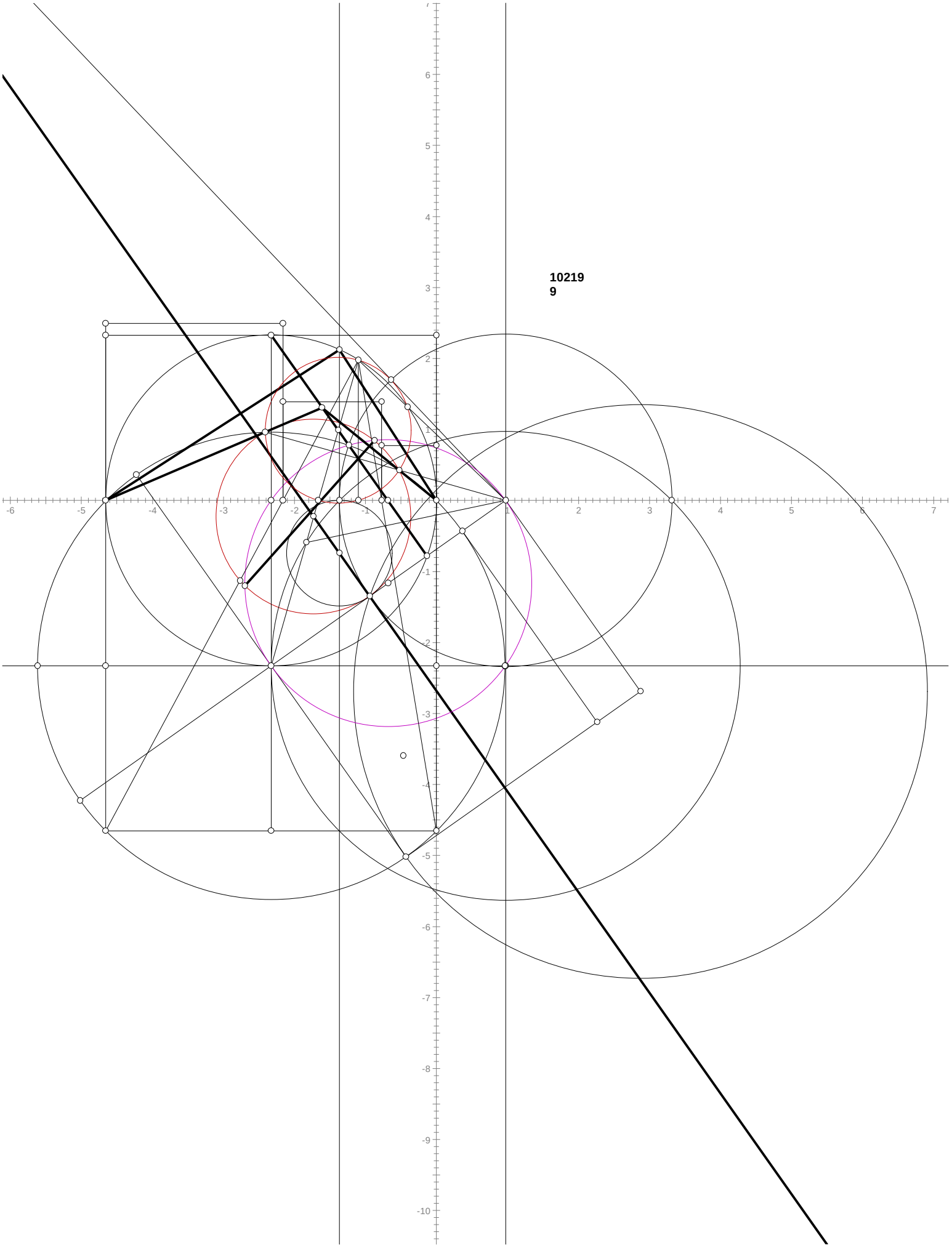

Pull Me

Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.



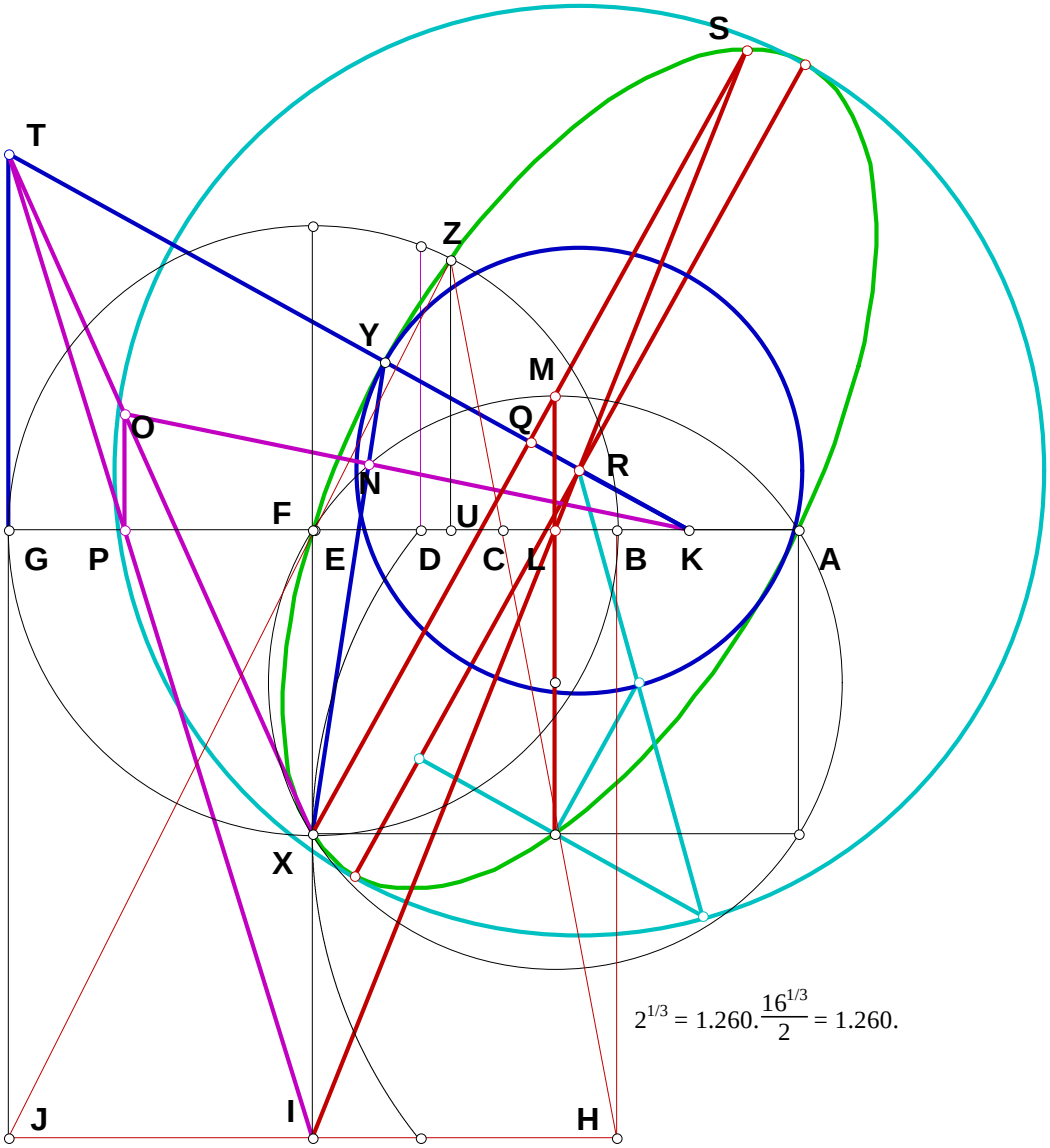
$\frac{AB}{AC}$ = 14.000

Linear Multiplication and Division,
Dual Vanishing Point Method
Unit and Universe of Discourse.


10219
9

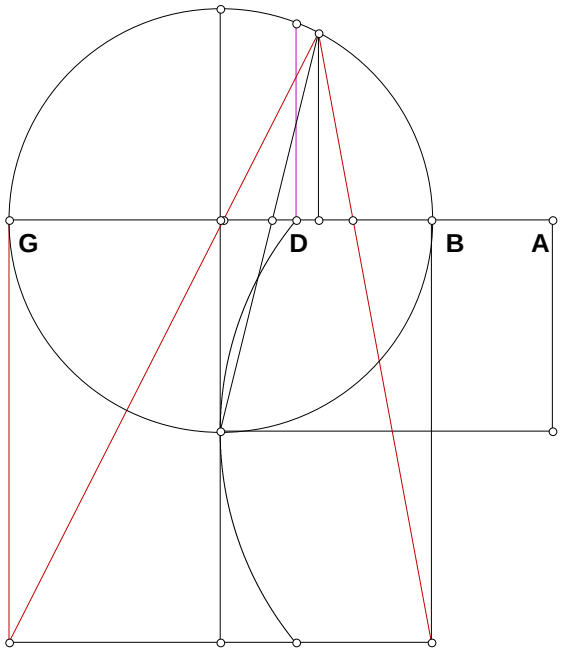
Appendix A

Step by Step



As this method of construction does not quite make it to two, sixteen will do as well.

Plate 1



$AB = 1, N = 16, AG = AB \times N$

$BG = AG - AB, AD = \sqrt{AB \times AG}$

Plate 2

$BF = \frac{BG}{2}, FG = BF, AF = AB + BF$

$FX = BF, Mf = \frac{\sqrt{AF^2 + FX^2}}{2}, LF = \frac{FX}{2}$

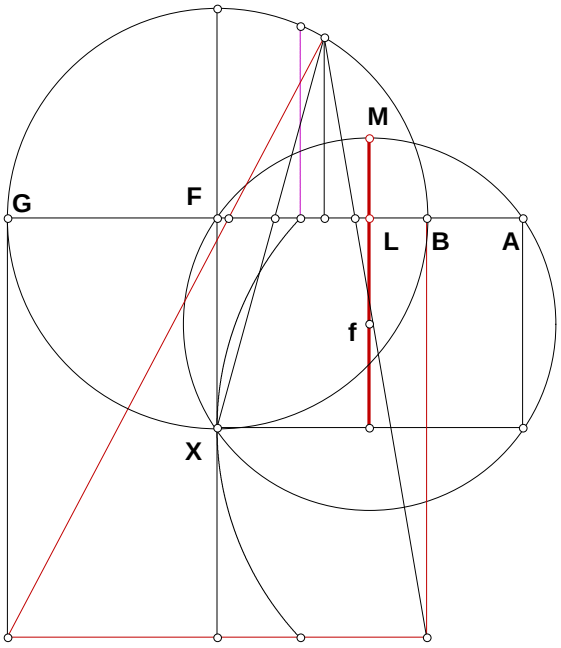
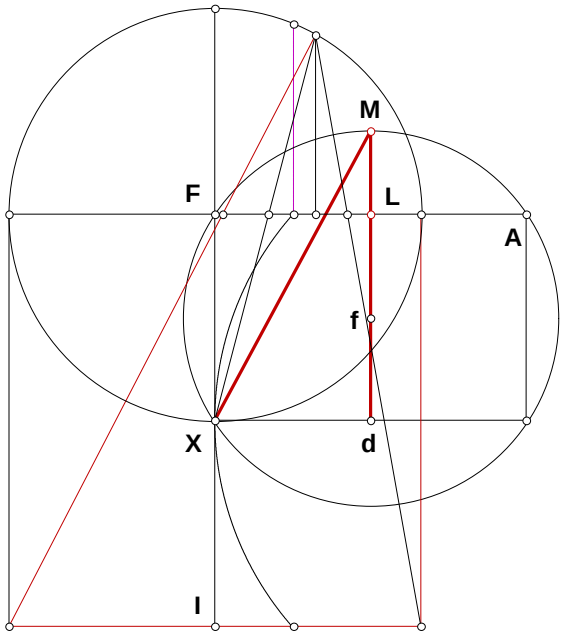


Plate 3



$$ML = Mf - Lf,\ FL = \frac{AF}{2},\ Xd = FL,\ df = Lf$$

$$IX = FX,\ Md = Mf + df,\ MX = \sqrt{Xd^2 + Md^2}$$

Plate 4

$$SX = \frac{MX \times IX}{IX - ML},\ Lg = \frac{FL \times ML}{ML + FX},\ QX = \frac{SX}{2}$$

$$Fg = FL - Lg,\ Xg = \frac{MX \times Fg}{Xd},\ Qg = QX - Xg$$

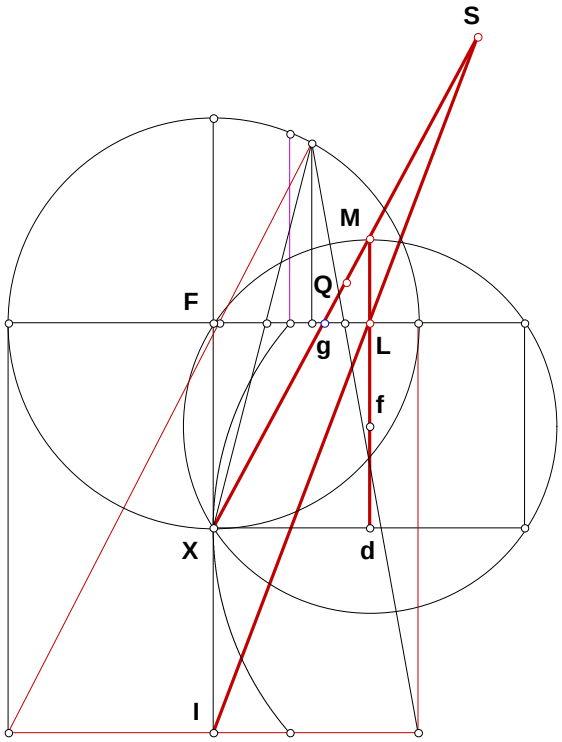
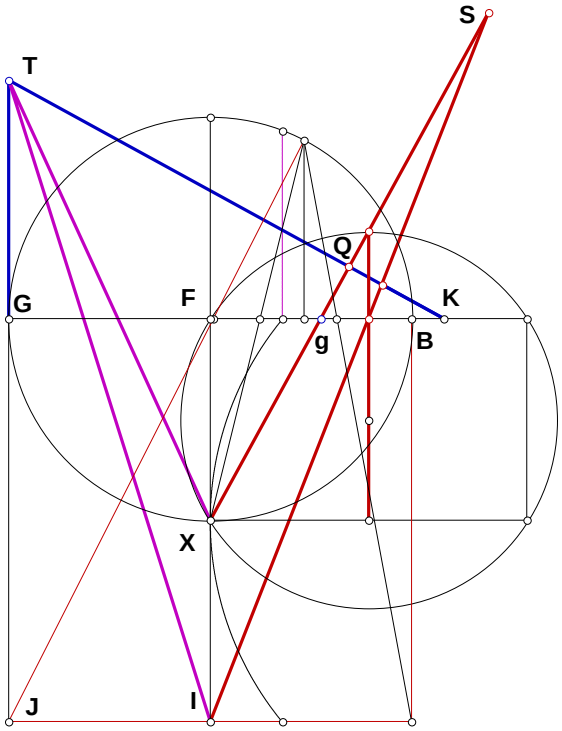


Plate 5



$$Kg = \frac{Xg \times Qg}{Fg},\ GK = FG + Fg + Kg,\ GJ = BG$$

$$GT = \frac{Fg \times GK}{FX},\ JT = GJ + GT,\ IJ = BF$$

Plate 6

$$FP = \frac{IJ \times GJ}{JT}$$

$$OP = \frac{IX \times GT}{JT}$$

$$KP = Fg + Kg + FP$$

$$Pi = Lf$$

$$Oi = OP + Pi$$

$$hi = \frac{KP \times Oi}{OP}$$

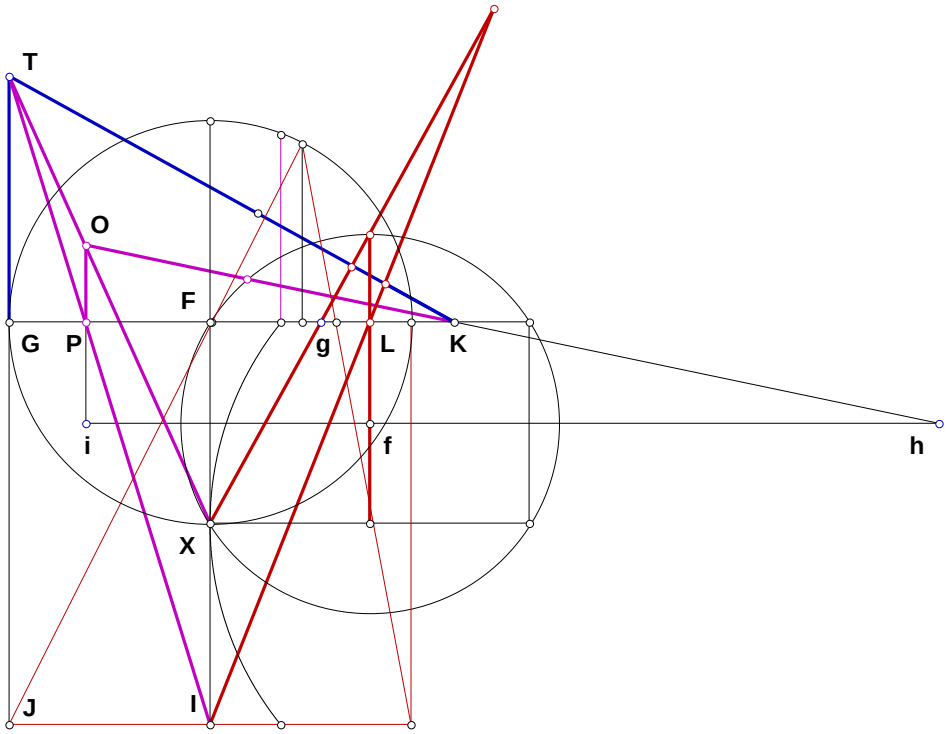
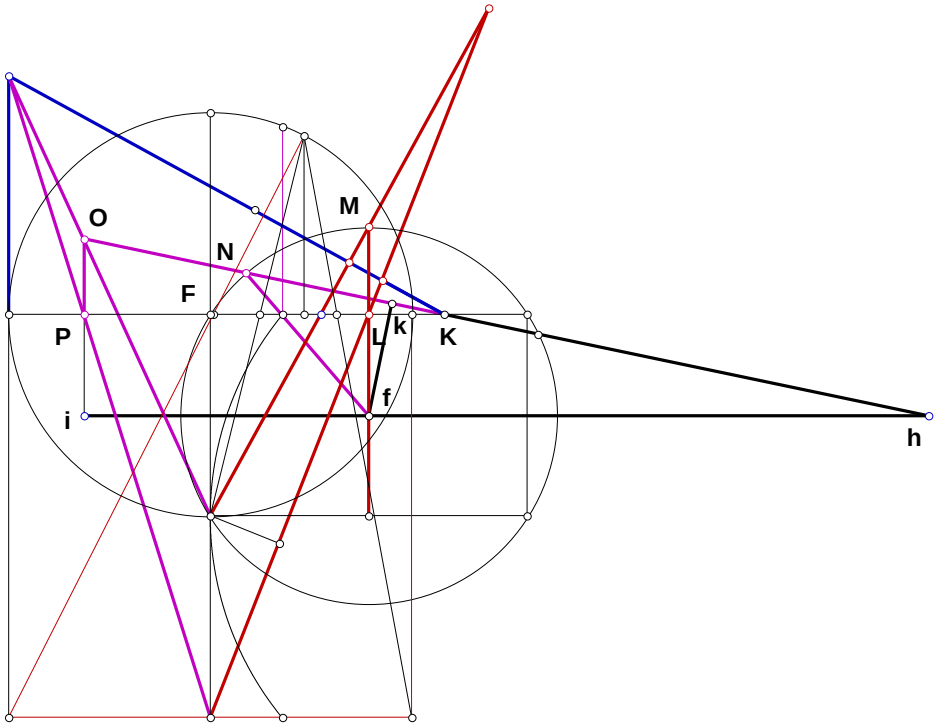


Plate 7



$fi = FP + FL$

$fh = hi - fi$

$KO = \sqrt{KP^2 + OP^2}$

$hk = \frac{KP \times fh}{KO}$

$fk = \frac{OP \times fh}{KO}$

$Nf = Mf$

$Nk = \sqrt{Nf^2 - fk^2}$

Plate 8

$Nh = hk + Nk$

$Oh = \frac{KO \times Oi}{OP}$

$NO = Oh - Nh$

$KN = KO - NO$

$NI = \frac{OP \times KN}{KO}$

$KI = \frac{KP \times KN}{KO}$

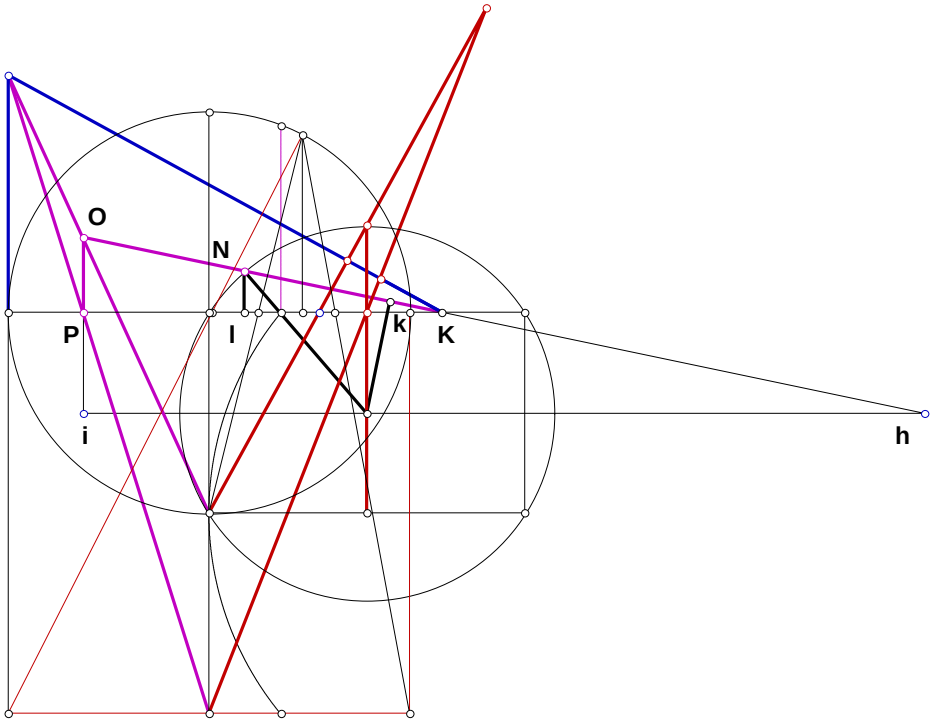
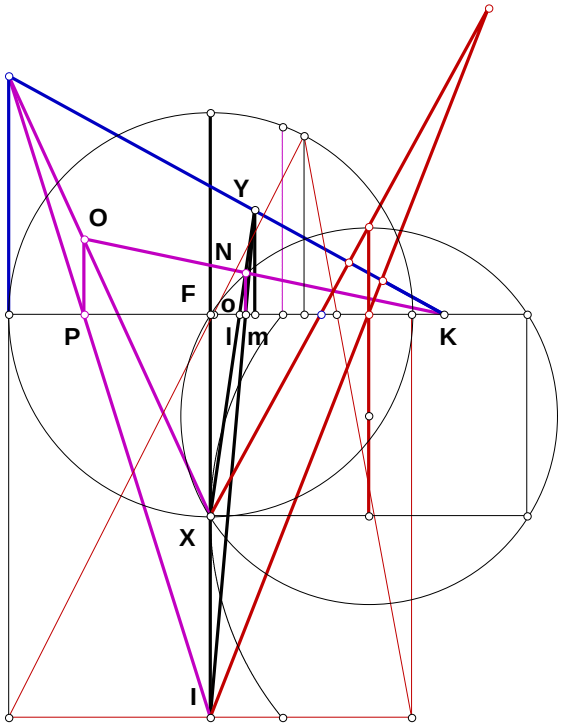


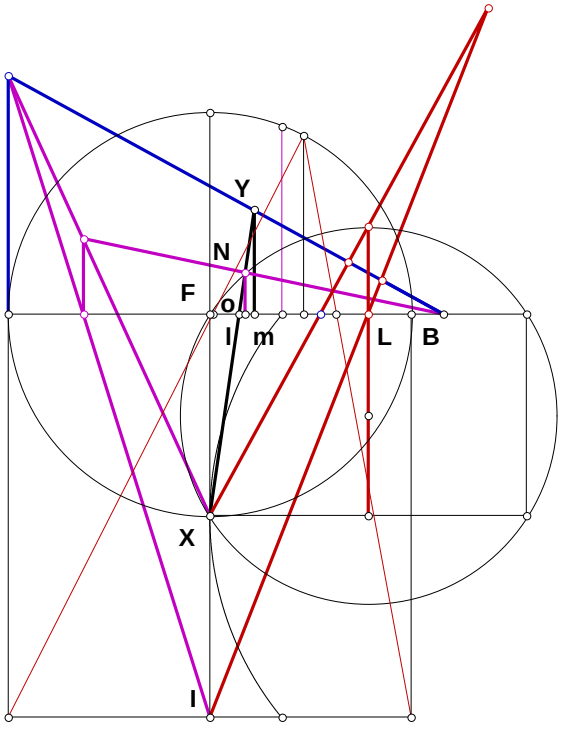
Plate 9



$$FK = KP - FP,\ Fl = FK - Kl,\ NX = \sqrt{(FX + Nl)^2 + Fl^2}$$

$$XY = \frac{NX \times IX}{IX - Nl},\ Fm = \frac{Fl \times XY}{NX},\ Km = FK - Fm$$

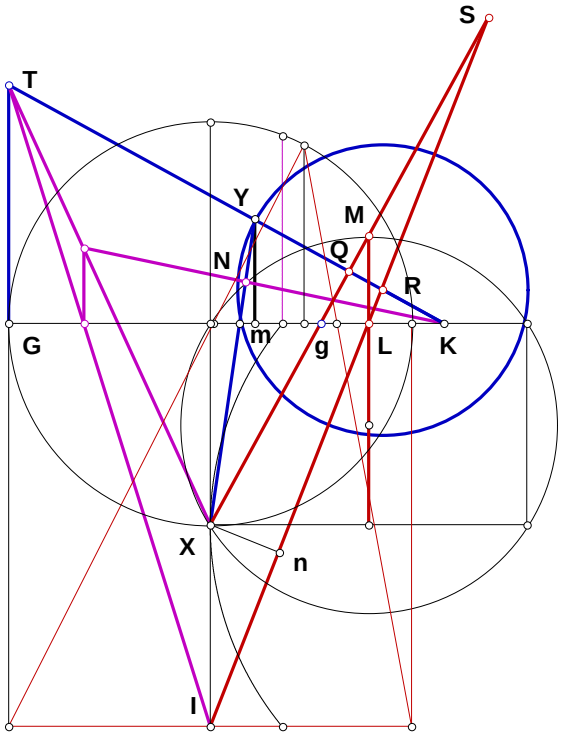
Plate 10



$$Fo = \frac{Fl \times FX}{FX + Nl},\ Xo = \frac{NX \times Fo}{Fl},\ mo = Fm - Fo$$

$$Ym = \frac{FX \times mo}{Fo},\ FI = 2 \times BF,\ IL = \sqrt{FL^2 + FI^2}$$

Plate 11 The Minor Axis



$$IS = \frac{IL \times IX}{IX - ML},\ In = \frac{IS^2 - SX^2 + IX^2}{2 \times IS}$$

$$Xn = \sqrt{IX^2 - In^2},\ QR = \frac{Xn \times QX}{IS - In}$$

$$KT = \sqrt{GK^2 + GT^2},\ KY = \frac{KT \times Ym}{GT}$$

$$KQ = \frac{GK \times Qg}{GT},\ RY = KY - KQ + QR$$

Plate 12

$$Xd = FL, dp = \frac{GK \times Xd}{KT},\ pq = QR$$

$$dq = dp - pq,\ Xp = \frac{GT \times Xd}{KT},\ Op = QX - Xp$$

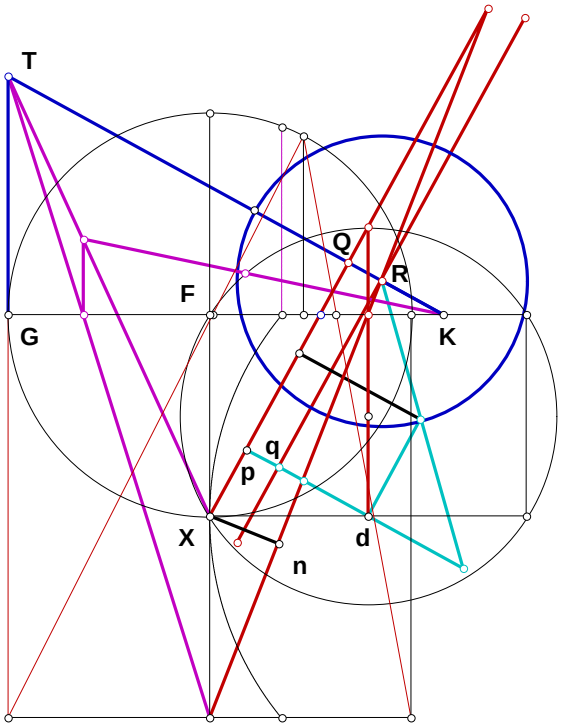
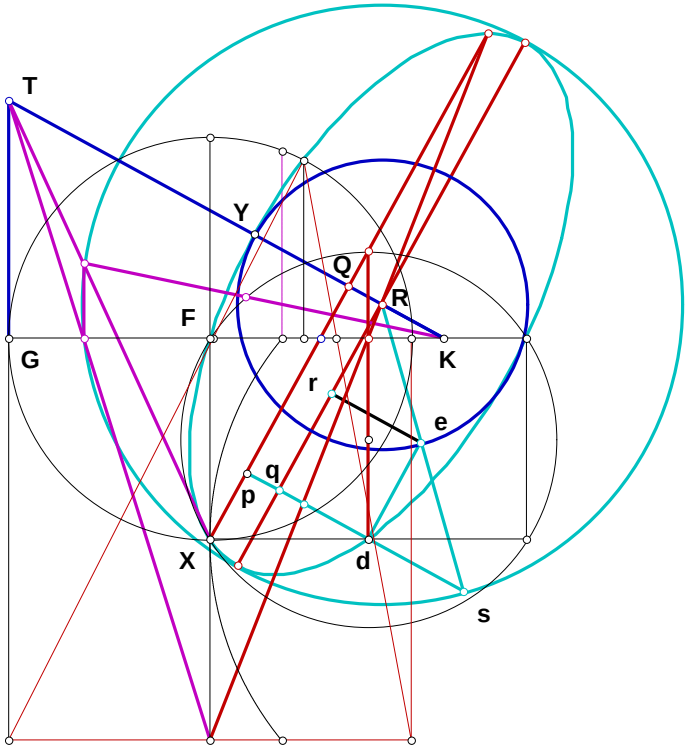


Plate 13 The Major Axis



$er = dq,\ Re = RY,\ Rr = \sqrt{Re^2 - er^2},$

$Rq = Qp,\ Rs = \dfrac{Re \times Rq}{Rr}$

Plate 14

$AC = (AB^2 \times AG)^{1/3},\ AE = (AB \times AG^2)^{1/3}$

$BC = AC - AB,\ BE = AE - AB,\ CE = BE - BC$

$EG = BG - BE$

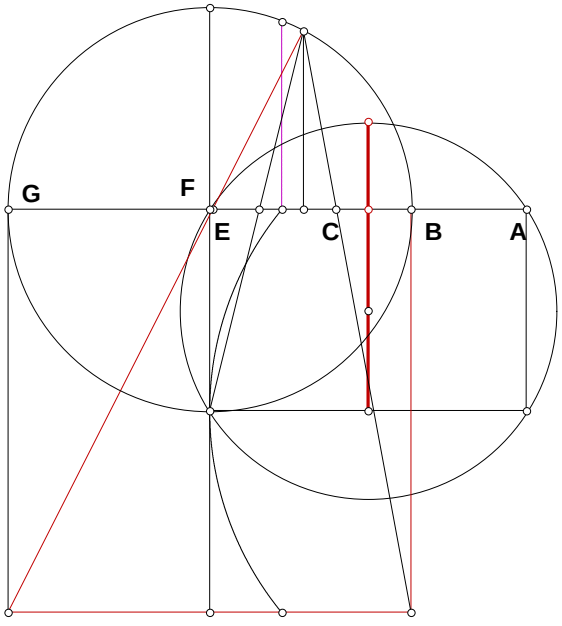
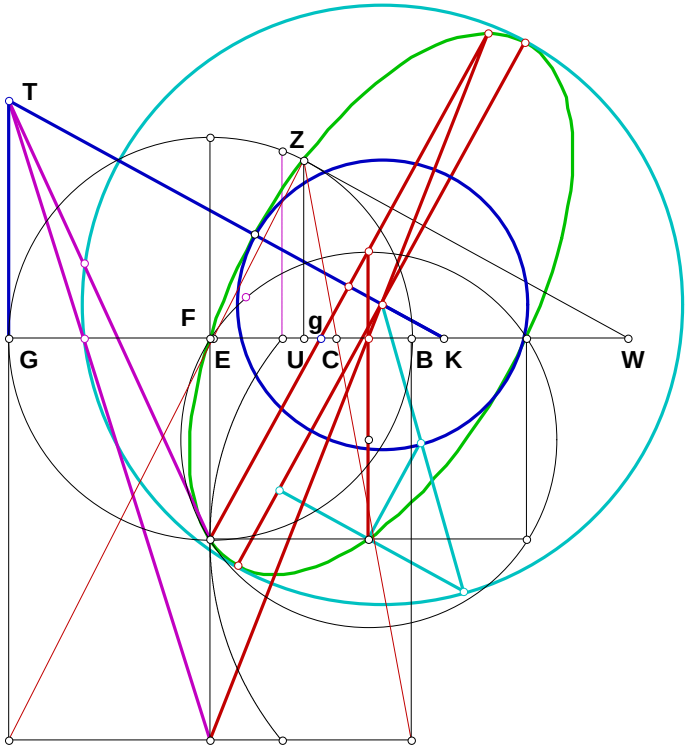


Plate 15



$$CU = \frac{BC \times CE}{BC + EG},\ BU = BC + CU$$

$$GU = BG - BU, UZ = \sqrt{BU \times GU}$$

$$UW = \frac{GK \times UZ}{GT},\ Gg = GK - Kg$$

Plate 16

$$tu = QR,\ gt = \frac{KT \times tu}{GK},\ Gt = Gg + gt$$

$$GW = GU + UW,\ Wt = GW - Gt$$

$$tv = \frac{GT \times Wt}{KT},\ Kt = GK - Gt$$

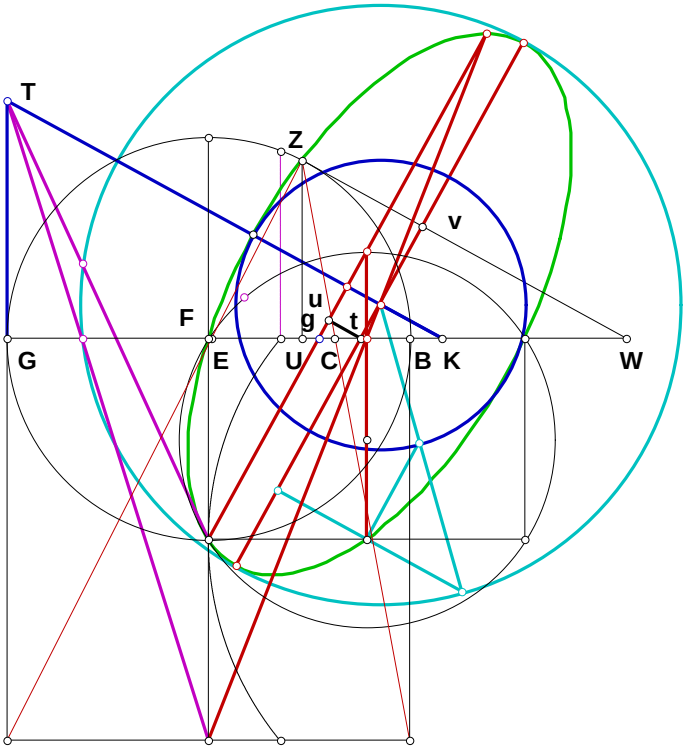
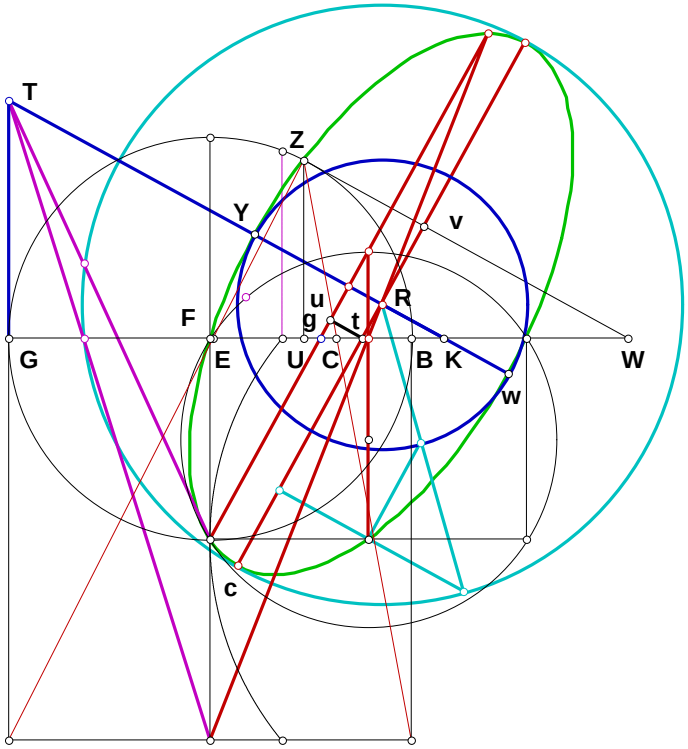


Plate 17



$$Rt = \frac{GT \times Kt}{KT},\ Rv = tv - Rt$$

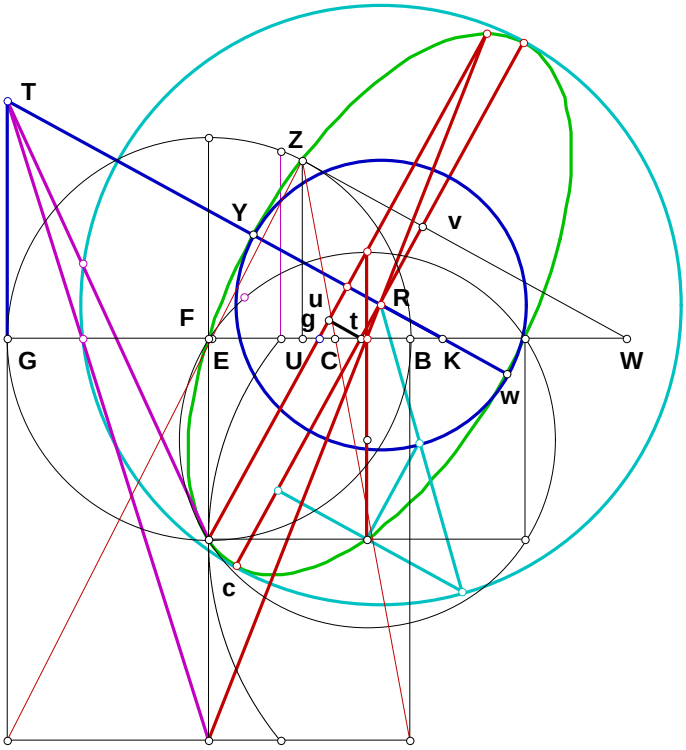
$$bc = 2 \times Rs,\ Rc = Rs,\ cv = Rc + Rv$$

$$Yw = 2 \times RY$$

Plate 18

$$WZ = \frac{KT \times UZ}{GT},\ Wv = \frac{GK \times tv}{GT}$$

$$Zv = |WZ - Wv|$$

$$N_1 = Yw,\ N_2 = bc,\ N_3 = cv,\ N_4 = bc$$

$$\sqrt{N_3 \times (N_4 - N_3)} \times \frac{N_1}{N_2} - Zv = 0$$